

Maria Palumbo s143106

Greenfield planning of Nordhavn area: An optimal design of the power distribution network

Master thesis, August 2018

Supervisors:

Senior Researcher Shi You, DTU

PhD student Sergey Klyapovskiy, DTU

Senior Data Analyst Carsten Buhl Nielsen, Radius



Chapter 1

Aknowledgements

I would like to show my gratitude to all the people that have made a difference during this period in which this project was produced. I would like to thank my supervisors Senior Resercher Shi You for the support and the kindness he has shown during the project. I would also like to show my gratitude to my other supervisors: Senior Data Analyst Carsten Buhl Nielsen and to PhD student Sergey Klyapovskiy, for the patience, the assistance and the interest shown for my project.

Abstract

This master thesis project focuses on designing 10 kV grid topology solutions for an urban green field area in Nordhavn, Copenhagen, Denmark. Nordhavn is a brand new district, being built from scratch, which presents a unique opportunity to work on the interface between architecture, buildings, infrastructure, the needs of the future residents and the requirements placed on a flexible energy system. Energylab Nordhavn is a full-scale urban lab, in which the City of Copenhagen tests solutions for electricity, heating, energy-efficient buildings and transport.

Today, replicating the existing topology solutions for new areas with some changes is a general approach adopted by the network designers; however, in many cases, the derived results usually are not close to be optimal. Instead of following the existing design guideline, mathematical modeling and optimization techniques will be developed and applied to find new topology solutions. The optimization will consider different criteria such as the customers' electric power consumption, the geography of the area, the type of cables used, the reliability rates and most importantly, the total cost of the network. The main tools used are Matlab, Excel and Google Maps. The main objective of the optimization is to minimize the total cost and maximizing the reliability of the electrical distribution system.

The report will first include in a literature review part where the optimization problem is presented Chapter 2 together with its formulation and the approaches used to solve it in Chapter 3. The different solutions will be presented, analyzed considering different input data in 4 .

The report will include a section about the future or further considerations and implementations concerning the topic in general and about Nordhavn specific case in Chapter 6. Additional plots and tables will be shown in Appendix A and B .

Contents

1	Aknowledgements	ii
	Contents	iv
2	Introduction	1
2.1	Report Overview	4
3	Algorithm	8
3.1	Theoretical Optimization Problem	8
3.1.1	Case 1 and 2	9
3.1.2	Case 3 and 4	11
3.1.3	Case 5	14
3.2	Building the Algorithm	15
3.3	2-Degree Vertex Constraint	16
3.4	3-Degree Vertex Constraint	20
4	Testing the Topology	22
4.1	Case 1 : Minimal Length - 2 Edges	22
4.2	Case 2 : Minimal Length + Cable Size - 2 Edges	22
4.3	Case 3 - Minimal Length + Cable Size + Reliability - 2 Edges	27
4.4	Case 4 - Minimal Length + Cable Size + Reliability + N-1 - 2 Edges	29
4.5	Summary	30
4.6	Case 1 to Case 4 - 3 Edges	31
4.7	Summary	34
5	Chapter 4	36
5.1	Case 5 : Time Periods	36
5.2	Case 4 : Christmas	37
5.3	Case 4 : Summer Vacation	38
5.4	Case 4 : New Year's Eve	39
5.5	Case 4 : January	40
6	Conclusion	42
6.1	Future work	44
	Bibliography	45
	Appendices	46

A Tables

47

B Plots

63

Glossary

- **ART:** Actual repair time
- **ARST:** Actual re-supply time
- **DCMST:** Degree Constrained Minimum Spanning Tree
- **MS:** Main Substation
- **MST:** Minimum Spanning Tree
- **MTBF:** Mean Time Before Failure
- **NNA:** Nearest Neighbor Algorithm
- **SAIDI:** System Average Interruption Duration Index
- **SAIFI:** System Average Interruption Frequency Index
- **TSP:** Travelling Salesman Problem

Chapter 2

Introduction

The *electricity grid* is the term used to refer to an interconnected network for delivering electricity from the *suppliers* to the *consumers*.

There are several motivations that drive the requirement to move on from the traditional power grid. The first concern is the reduction of greenhouse gas emissions. A second motivation is the inflexibility of the current power grid, that can't support the development of renewable energies or other forms of technologies that would make it more sustainable. A third reason is that due the transformation that the energy market is undergoing, a small-scale approach is considered beneficial to the electricity system in many ways: from reduced losses, since the electricity source and load are closer, to system modularity, to smaller investments compared to large-scale energy solutions. In particular, the fact that renewable sources such as wind and solar are intermittent poses a significant problem for an inflexible grid that does not spread information to control centers rapidly and thus cannot increase other sources of production. All of these problems are addressed by the smart grid through improved communications technology, (ICT), with numerous benefits for both the supply and demand sides of the electricity market, in order to improve the reliability, security, and efficiency of the electrical grid. [6]

An electrical grid consists of three distinct element:

- 1. Generation:** Power stations where the electricity is generated.
- 2. Transmission:** Transfer of the power.
- 3. Distribution:** Delivery to the consumer.

For the grid to function effectively these three elements have to be in balance at all times.

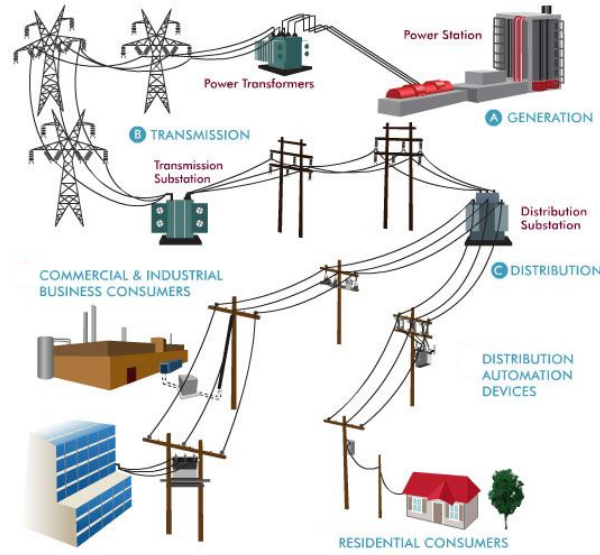


Figure 2.1: Electrical power grid scheme

[1]

Electricity is generated in power stations which to convert the mechanical energy of a turbine into electrical energy by the use of a generator. The solutions are usually located near a source, and are often very large and far from densely populated areas.

The power transmission network moves power over long distances, sometimes across international borders, until it reaches its wholesale customer. In this second phase the electricity is "stepped-up": the voltage is increased, with a proportional decrease of electric current. Step-up transformers are used is because when travelling long distances through a conducting wire, electricity will inevitably lose energy due to resistance. This problem is largely solved by the use of high voltage power lines in order to minimize the power loss due to resistance.

Given the electric power P

$$P = IV$$

where V is electric potential or voltage and I is electric current, Ohm's law states that voltage across a resistor is linearly proportional to the current flowing through it, or

$$V = IR$$

Assuming I as as a constant and replacing V with IR we get that power is proportional to resistance

$$P = I^2R,$$

therefore the power loss in the transmission line is be proportional to I^2 . [7]

On arrival at a substation, the power will be "stepped-down" from a transmission level voltage to a distribution level voltage. An electrical power substation is therefore a conversion point between transmission level voltages and distribution level voltages. As the power exits the substation, it enters the distribution wiring. Finally, upon arrival at the service location, the power is stepped down again from the distribution voltage to the required service voltages.

This project main focus is the Distribution grid phase, the terminal part of the power grid infrastructure

that reaches the end user, and specifically the network topology design.

Planning of distribution grids is often limited by the existing topology and design used in the area. Since the current topology may not necessarily represent an optimal solution and be cost-efficient, electrical utilities are interested in new approaches when designing their systems, specially when starting to electrify new areas where no such limitations exist.

Planning exercises can be divided into three categories:

Greenfield planning refers to the planning of new areas, where no previous electrical infrastructure exists. The input data could be in a form of load centers that should be supplied with power. Greenfield projects give the most flexibility to the distribution grid planner in terms of topology and layout selection, but are not common tasks in most countries.

The second type of planning is distribution *grid extension*. Expansion assumes that the existing network is just extended to a new area (where some electric infrastructure can already be located). In some cases a new area can be supplied from the main substation located somewhere in the old grid. Since the new area is a part of the bigger distribution grid (like a district is a part of a city), the limitations of the bigger network (existing ND and OS) is imposed in such planning.

Lastly, *grid reinforcement planning* is not necessarily aiming at introducing new elements in the already existing grid, but rather on updating the old ones (transformers, cables), when they are unable to deliver their services anymore. This is the planning exercise with the most limitations.

It should be noted, that the proposed division is not fixed, since in some cases the difference between different types could be very small.

This master project focuses on greenfield planning in order to propose optimal topology for the distribution system of Nordhavn, wherein pros and cons of different design criteria both technical and non-technical and alternative grid planning solutions are evaluated. Unlike grid reinforcement and grid extension approaches, a greenfield project is not constrained by prior work, and it doesn't require to remodel or demolish an existing structure.

2.1 Report Overview

All the data used in this report were provided by Radius.

The Danish power distribution company Radius was renamed from DONG Energy El Distribution to Radius Elnet on 1 April 2016 and was given its own distinct visual identity to make it easier for the Danish customers to distinguish between the power distribution company and the rest of DONG Energy.[5] Ørsted A/S (formerly DONG Energy) is the leading energy company in Denmark.

The topology currently in use in Nordhavn consists of 59 Substations; the Main Substation (MS) represents an additional node from which all the connections must originate from. An overview of the actual positions of the nodes are shown in Figure 2.2, together with the actual topology.

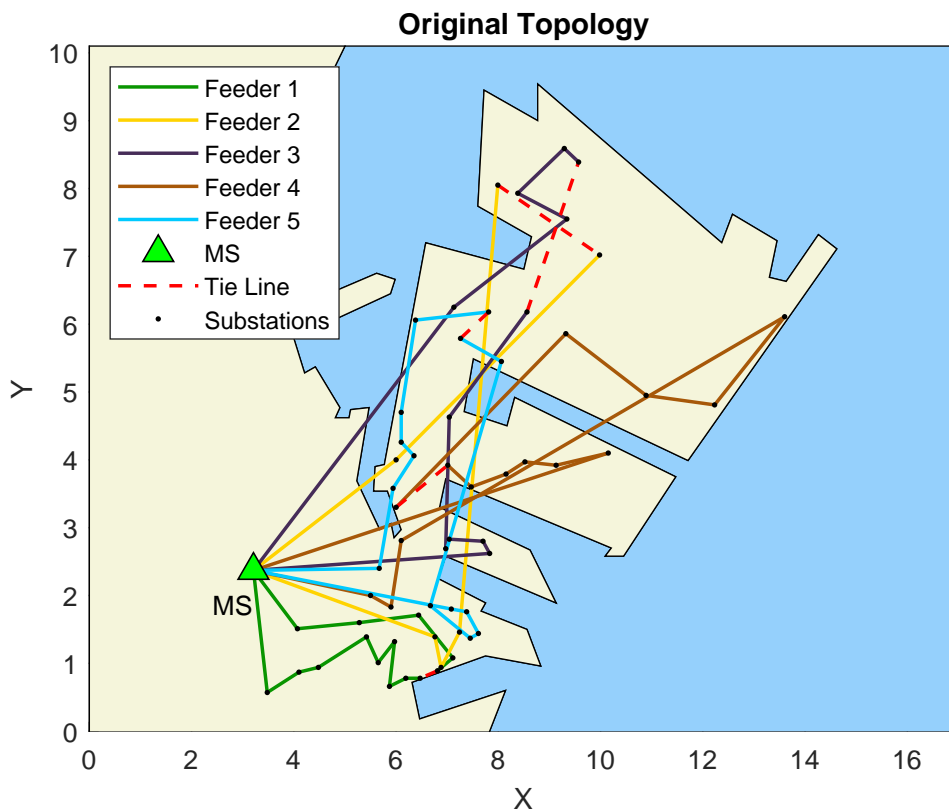


Figure 2.2: Pre-existing topology

The main purpose of the optimization algorithm will be to minimize the costs and at the same time provide a feasible and reliable design for a new topology. The main criteria that will influence the total cost are:

- Type of cable connecting the substations.
- Length of the cables.
- Consumptions.

The constraints that apply to all the criteria of selection of the optimal topology are:

- Constrain 1: Start of cable connection from the MS ▲.

- Constrain 2: Geography of the area.

The criteria used for selection are:

1. Distance between substations and thus length of the cables.
2. Cable type.
3. Reliability Index.
4. N-1 analysis.

The algorithm will be tested using fewer nodes with regards to the original topology, namely 44. The new disposition of the nodes is shown in the Figure B.13.

The hourly electrical power consumption for each substation collected during the one year period 2006, namely 8760h. The demand data and the number of installations per substation were provided by Radius. Table 2.1 shows position, consumption and cable type used for the single connection. More specifications will be added in the next section.

Since the only data provided about the actual distances of the substations were related to the original topology (such as the length of the cable between two connected substations) it has not been possible to use either the real coordinates or the real distances in the analysis. Therefore when referring to distance it implies unit of length instead of km. Using the Nordhavn original system and Google Maps made it possible to recreate a faithful version of the grid and the positions of the substation.

Index	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cable
1	1	2	3.21	2.37	3.48	0.57	108.9415	3.9118	1
2	2	3	3.48	0.57	4.1	0.87	104.9991	1.3537	1
3	3	4	4.1	0.87	4.48	0.94	57.6091	0.70214	1
4	4	5	4.48	0.94	5.42	1.39	288.6599	2.2937	1
5	5	6	5.42	1.39	5.65	1.01	64.5053	1.1721	1
6	6	7	5.65	1.01	5.97	1.32	96.735	1.6525	2
7	7	8	5.97	1.32	5.87	0.66	84.9123	3.2886	2
8	8	9	5.87	0.66	6.19	0.78	17.5464	3.5254	2
9	9	10	6.19	0.78	6.47	0.78	252.3816	9.238	1

Table 2.1: Data sample from the original topology

Where:

- Index : Connection label.
- Sub_i : Substation label.
- X_i, Y_j : XY-coordinates of each node/substation.
- Peaks : Yearly peak of consumption.
- Cables : Type of cable used.

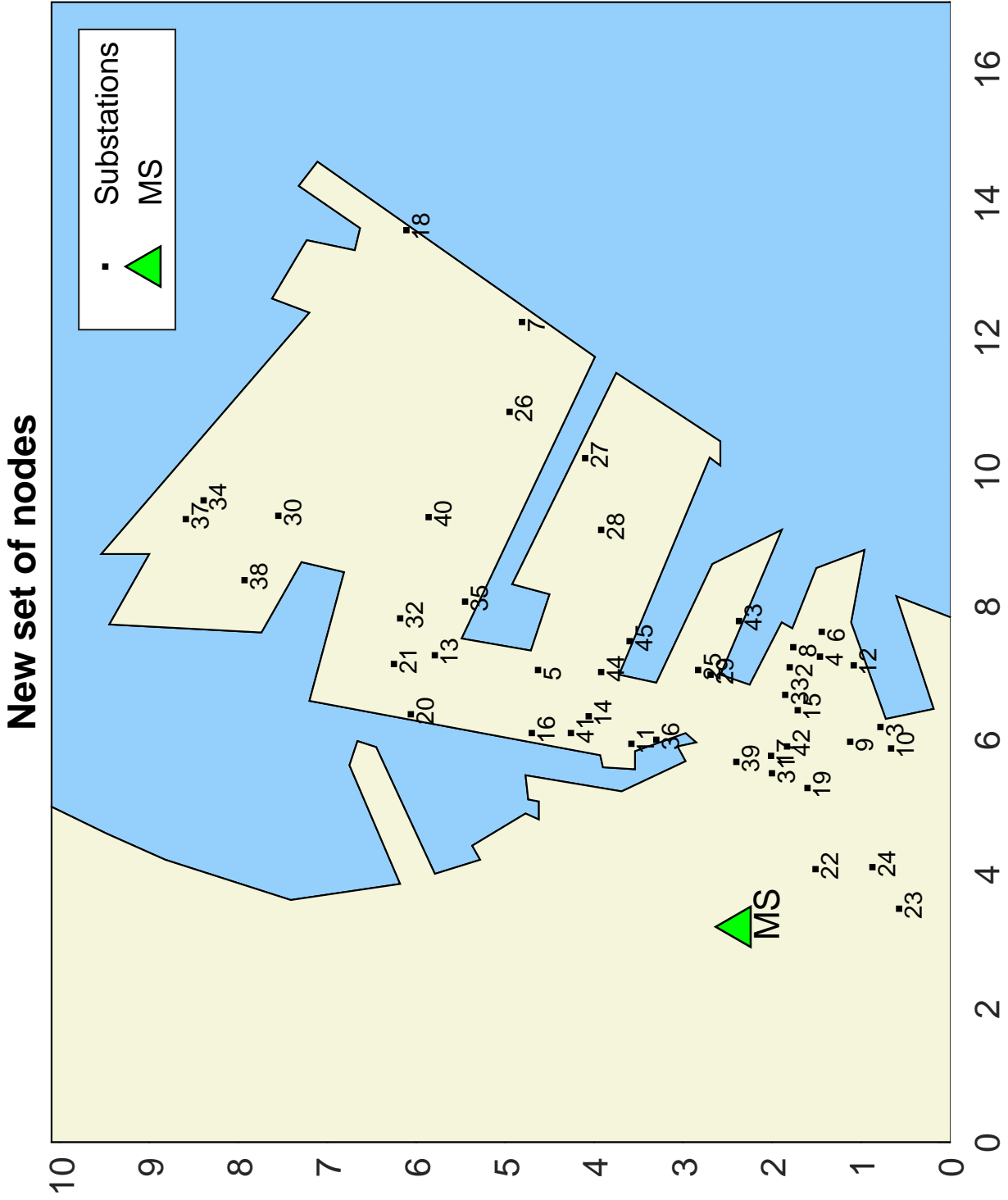


Figure 2.3: New configuration of the substations

The approach to the topology optimization problem consists of

1. Designing the algorithm:

Mathematical formulation of the constraints

- Minimal length

- Minimal length + Cable size

- Minimal length + Cable size + Reliability

- Minimal length + Cable size + Reliability + N-1

- Voltage test

2. Case study 1 - Testing the topology.

First assumption: all the peaks occur at the same time.

3. Case study 2 - Testing the topology.

Second assumption: peak demands are non-coincident. Different time scenarios.

4. Conclusion

Conclusion

Future work.

Chapter 3

Algorithm

3.1 Theoretical Optimization Problem

The formulation of the problem recalls the mathematical formulation used in linear integer programming for routing or scheduled related problems.

A network and more specifically a power grid can be seen as a system of roads: there are many ways to get from any given node to another, so if there is an accident there are still many ways to reach the final destination. In the same way there are other paths for the electricity to get to the consumer again in case one connection is interrupted.

Network science has its basis in graph theory and in the mathematical branch that studies topology. When referring to networks, topology is defined as the particular pattern of the connections between the nodes that compose the network.

The variables and constants used in this problem are:

- \mathbf{i} (as for \mathbf{j}) : index that represent a node and refers to a specific substations;
substations are labeled with the numbers $1, \dots, N$
 $\mathbf{i} = 1$ represents the MS, $2 \leq \mathbf{i} \leq n$ the other substations.
- \mathbf{c}_{ij} : cost of connecting substation \mathbf{i} to substation \mathbf{j}
- \mathbf{x}_{ij} : distance between substation \mathbf{i} and substation \mathbf{j}
- \mathbf{N} : number of substations
- \mathbf{l}_i : demand of the substation
- \mathbf{L}_{\max} : maximum load.
- \mathbf{R}_{\min} : minimum reliability rate.

$$\sum_{i \neq j}^N c_{ij} \cdot x_{ij} \quad (3.1)$$

$$\sum_{\substack{i=1 \\ i \neq j}}^N x_{ij} = 1, \quad 1 \leq i \leq N \quad (3.2)$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N x_{ij} = 1, \quad 1 \leq j \leq N \quad (3.3)$$

$$\sum_{i,j}^N l_{i,j} \leq L_{\max} \quad (3.4)$$

$$\sum_{i,j}^N rRate_{x_{ij}} \leq R_{\min} \quad (3.5)$$

$$\text{Case 1: } \deg(i) = 2 \quad (3.6)$$

$$\text{Case 2: } \deg(i) = 3$$

(3.1) : The objective of the optimization problem is to minimize the sum of the costs of all the connections.

(3.2)&(3.3): Ensures that every substation is served only once.

(3.4): The total load of the substation should not exceed the capacity of the cable, that is the maximum wattage (load) L_{\max} that the cable can be carried.

(3.5): The reliability index should not be lower than R_{\min} .

(3.6): The number of edges connected to the same node cannot exceed $\deg(i)$.

The topology found will be represented by the vector \mathbf{W} of ordered indexes.

3.1.1 Case 1 and 2

- Minimal Length

The aim is to find the minimal length path connecting all the nodes; in this scenario the cables connecting the substations are considered equal despite the differences in voltages and current carried; the total cost therefore depends only on the length of the network.

The total cost is given by:

$$\text{TotalDistance} = \sum_i^N d_i(\mathbf{W}(i)),$$

$$\text{TotalCost} = \text{TotalDistance} \cdot \text{Cost},$$

where Cost is the cost per unit of length (km), and W the nodes/substations indexes in order of connection.

Initially, only one link will connect to the MS. In later analysis, more feeders will be introduced.

- Cables Category

Four different kind of cables are considered: $C = (C_1, C_2, C_3, C_4)$, that can carry:

Cable	I_{max} [A]	S [MVA]
1	234	4.07
2	280	4.85
3	361	6.25
4	401	6.95

Where given the current I the power S is found as:

$$S = \sqrt{3} V_{line} I_{line}$$

To each type of cable a different price is assigned: $Cost_1, Cost_2, Cost_3, Cost_4$. The cable category is assigned by taking into account the consumption peak for each substation, the sum of the consumption peaks of the substations connected and the maximum capacity that can be carried by the cable.

- Price of the cable per unit of length has been set to:

$$Cost_1 = 550000 \text{ Dkk}$$

$$Cost_2 = 1.3 \cdot Cost_1 = 715000 \text{ Dkk}$$

$$Cost_3 = 1.5 \cdot Cost_1 = 825000 \text{ Dkk}$$

$$Cost_4 = 1.6 \cdot Cost_1 = 880000 \text{ Dkk}$$

The cost of the new topology obtained by introducing the cable types is given by:

$$\text{TotalCostCab} = \sum_{k=1}^N (\text{distance}_k \cdot \text{Cost}_k); \quad (3.7)$$

where N is the number of substations and the vector *distance* represents the pairwise distances between the consecutive substations (therefore of the single cable connection between two consecutive substations) and *Cost* represents its cost such that $\text{Cost} \in [Cost_1, Cost_2, Cost_3, Cost_4]$.

- Cable Capacity

L_{max} is set as the maximum capacity load in the system. The cable category is selected according to the capacity that can be carried by the specific type of cable, then the original cable connection is changed by the algorithm according to it. As shown already in Equation 3.4 the sum of the loads connected should not exceed

$$\sum_{k=1}^N l_{i,j} \leq L_{max} = \max(\text{Loads}) \cdot K$$

where $K > 1$ can be arbitrarily chosen.

The total cost is evaluated using the same formula (3.7), but in this case the resulting topology will most likely consist in more than one feeder.

The expected outcome from this first analysis is that that increasing the selection criteria the total length and the total cost of the resulting topology will increase.

In addition the topologies resulting from Case 1 and Case 2 aren't expected to be good candidates as optimal topology since a long line usually implies a low reliability of the system.

3.1.2 Case 3 and 4

- Reliability Index - SAIDI SAIFI

Reliability is the most important characteristic of quality in most power delivery planning. The introduction of the Reliability Index plays an important role since it defines the probability of success as the frequency of failures, namely

$$P(\text{Reliability}) = 1 - P(\text{Failure}). \quad (3.8)$$

In our specific case the Reliability is related to the single cable connecting two adjacent nodes as well as to the totality of the connections: the more nodes are connected the more the total Reliability Index (rRI) decreases. The Reliability Index rRI is fixed at 0.99 for each km of cable. The individual Reliability Rate is calculated as:

$$\text{Rate}_i = \text{rRI}^{d_{i,j}}, \quad (3.9)$$

where $d_{i,j}$ is the length of the connection between the substations i and j .

Frequency and duration of interruptions are both important in determining the impact of service interruptions on customers and in determining the reliability of the grid. Consumer cost of reliability is evaluated by assessing the cost impact of power system outages on the consumer namely the cost of doing without power. Two of the indices typically used are SAIDI and SAIFI, defined as:

System Average Interruption Duration Index:

$$\text{SAIDI} = \frac{\text{sum of the durations of all customer interruptions}}{\text{total customers in system}}$$

System Average Interruption Frequency Index:

$$\text{SAIFI} = \frac{\text{number of customer interruptions}}{\text{total customers in system}}$$

Including the number of customers in the analysis is important since it might be pointless and too expensive to guarantee 99.9999% availability to a substation that provides power only to one consumer. Eventually it is relevant to consider the customer category as well since for some industries a five minute interruption is nearly as damaging to productivity as a one-hour interruptions, and for some other customers short outages cause no significant problem, but they experience inconvenience during a sustained interruption.

According to [5] the average duration of interruption is 0.5 h (SAIDI) with a frequency of 0.5 per year (SAIFI) [5] and considering an average re-supply time between 2 and 4 hours, the Reliability rate has been set to 99.98%, the chosen reliability rate translates in a average lack of electrical power of 1.625h per year. The expected SAIDI and SAIFI according to Radius are SAIDI 0.4–0.5 hours per year (h/y) and SAIFI 0.3 – 0.5 times per year(t/y). In this report the analysis of the SAIFI and SAIDI refers *only* to the probability of failure of the cables, thus doesn't include failure

of other components, therefore the SAIFI value should be smaller than the reported annual average presented above.

Given the vector Rates of the Reliability Indexes, the final reliability is set to be greater than

$$\text{tRr} \geq \text{minRI} \quad (3.10)$$

minRI, which is the least acceptable Reliability Index. Consequently any topology with a lower tRr is not acceptable and therefore it will be altered by the algorithm in order to meet the set constraint.

By considering different topology designs and solutions the expected outcome is that a system with a low reliability has low values of SAIFI and SAIDI.

One way to evaluate SAIFI and SAIDI is to consider the failure rate FR_{c_k} for each type of cable $j = 1, \dots, 4$:

Cable Type	Failure Rate (FR_{c_k}) Per Cable Type
1	0.0015
2	0.0020
3	0.0025
4	0.0030

The failure rate is calculated for each cable connection $\text{FR}_{i,j}$ and for each load FR_{l_i} .

$$\text{FR}_{i,j} = \text{FR}_{c_k} \cdot d_{i,j}, \quad \text{times/km/year} \quad (3.11)$$

FR_{l_i} is evaluated as the probability of

$$\text{FR}(l_i) = P(\text{FR}_{l_{i-1}}) + P(\text{FR}_{l_i}) - P(\text{FR}_{l_{i-1}}) \cdot P(\text{FR}_{l_i}), \quad \text{times/km/year} \quad (3.12)$$

where $P(\text{FR}_{l_{i-1}}) \cdot P(\text{FR}_{l_i})$ is so small to be negligible.

MTBF:	Mean Time Before Failure. defined as the predicted elapsed time between inherent failures of the system. $MTBF = \frac{1}{FR}$ hours/year
Actual repair time (ART):	from 8h to 16h (not N-1 secure).
Actual re-supply time (ARST):	from 0.5h to 4h (N-1 secure).
IFI:	Individual Frequency Index. $IFI = \frac{8760}{MTBF}$ times/year
IDI:	Individual Duration Index. $IDI_1 = \frac{IFI}{ART}$ hours/year $IDI_2 = \frac{IFI}{ARST}$ hours/year

and finally:

$$\mathbf{SAIFI} = \frac{\sum_i^N IFI_{l_i}}{\sum_i^N cN_i}, \quad \text{FREQUENCY: times/year} \quad (3.13)$$

$$\mathbf{SAIDI1} = \frac{\sum_i^N IDI1_{l_i}}{\sum_i^N cN_i}, \quad \text{DURATION: hours/year} \quad (3.14)$$

$$\mathbf{SAIDI2} = \frac{\sum_i^N IDI2_{l_i}}{\sum_i^N cN_i}, \quad \text{DURATION: hours/year.} \quad (3.15)$$

where cN is the number of customers supplied at each Substation.

- Cable Category and Reliability

Both Cable Category and Reliability Index are taken into account in order to select a topology. The aim is to select a topology with the minimum cost and a reliability within the expected range.

- Reserve cable and N-1 analysis

A contingency is defined as "an event such as an emergency that may but is not certain to occur". In power systems, a contingency is when an element of the electric grid fails, e.g., generator, transmission line, substation, transformer, etc. If a system is N-1 contingent, it means that the system can continue to operate within nominal limits if 1 element fails. Energy distribution companies have to reconnect areas affected by an outage within a very short time, and observe operational constraints, to minimize the possibilities of financial losses, that's why distribution networks should have more than one route to deliver energy to any node of the network. Switches in the network are opened to create the topology used in normal operation and in the case of an outage, alternative routes can activated by opening or closing switches located at specific points of the network. After the initial topology is established, reserve cables are added by the algorithm. The N-1 analysis should be performed in order to estimate, whether it's still possible to supply all load centers with power in case of outages. The type of cable for the reserve cable is set to C4

3.1.3 Case 5

- Different Time Scenarios

All the above criteria will be used in different scenarios:

- 1 The peaks of consumption for all substations occur at the same time;
- 2 The peaks of consumption for the different substations occur at different times.

The consumption data used was collected hourly during the one year period of 2006. The 1st scenario represents the worst possible case. The expected output of the algorithm will potentially generate a different topology given the set of consumption data from different time period as input and most likely will present a topology that is to be preferred to the one developed in the 1st scenario, when the consumption are at their maximum values. Considering different time scenarios different topology and most likely lower cost.

3.2 Building the Algorithm

The foundation of the algorithm is based on solving the Traveling Salesman Problem (TSP) and the Minimum Spanning Tree problem (MST); their solutions give the network a basic topology that will be shaped and altered in the process in order to satisfy additional constraints. Before formulating the algorithm it is necessary to introduce graph theory.

mathematics, graph theory is the study of *graphs*, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices, nodes, or points which are connected by edges. A graph may be *undirected*, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be *directed* from one vertex to another.

In the power grid case, the graph is undirected and the cost of the cables between the substations corresponds to the weights of the edges. [9]

Given an undirected and connected graph G , a tree is an undirected graph in which any two vertices are connected by exactly one path. A spanning tree T of G is a subgraph that is a tree which includes all of the vertices of G , with the minimum possible number of edges. A Minimum Spanning Tree of G is a tree that spans G ¹ and is a subgraph² of G , without any cycles (loops). The cost of the spanning tree is the sum of the weights of all the edges in the tree. [3] The degree of a graph G vertex V is the number of graph edges which touch V , and the maximum degree of a graph G is the maximum degree of its vertices.[9]

The *problem* of the TSP concerns finding the shortest path connecting all the cities that a traveling salesman has to visit each city exactly only in only one tour and then return to the starting point. TSP can be modelled as an undirected weighted graph, such that cities are the graph's vertices, paths are the graph's edges, and a path's distance is the edge's weight. It is a minimization problem starting and finishing at a specified vertex after having visited each other vertex exactly once. Often, the model is a complete graph, each pair of vertices is connected by an edge. If no path exists between two cities, adding an arbitrarily long edge will complete the graph without affecting the optimal tour.[9] In this case we are not interested in paths that are connected to the starting point (MS) in a loop, therefore the algorithm has a fixed starting node and an open end.

The solution of the basic TSP formulation is a graph of maximum degree 2, where each node is connected to no more than 2 edges. This configuration reflects the design of the pre-existing topology.

The basic MST formulation doesn't have a vertex degree restriction on the maximum degree of the graph. The Degree Constrained Minimum Spanning tree (DCMST) is a minimum spanning tree in which each vertex is connected to no more than d edges, for some given number d . The case $d = 2$ is a special case of the TSP, since a path visiting all points exactly once it's a special kind of tree.

In the following section TSP will be used in order to connect all the substations using the shortest and least expensive possible path where each node is connected to no more than 2 edges and 3-DCMST will be used to find a minimum spanning tree of maximum degree $k = 3$, namely the shortest and least expensive possible tree where each node is connected to no more than 3 edges.

¹that is, it includes every vertex of G

²every edge in the tree belongs to G

These two methods can be both used to find an optimal *way*, either a tree or a single route, to connect all the given nodes and therefore they will be used to define an initial topology.

3.3 2-Degree Vertex Constraint

The Nearest Neighbour Algorithm (NNA) is used to get a preliminary approximation of the shortest path. It is based on the idea of optimizing the path choosing at each iteration the nearest node to the previous one until all the nodes have been checked. These are the steps of the algorithm:

1. Choose a starting node as origin,
2. Find the closest unvisited node to the origin;
3. Set that current node as new starting node, and mark it as visited;
4. Go to step 2;
5. Iterate until all the nodes have been visited, then terminate.

Since the NNA is a greedy algorithm, meaning that it performs the locally optimal choice at each iteration with the intent of finding a global optimum, it does not usually provide the optimal solution. For this reason it's used only to create an input for the 2-opt heuristic algorithm. A path P is called 2-optimal if there is no 2-adjacent path to P with lower length/cost than P.

The 2-opt heuristic algorithm checks for an adjacent path with shorter total length than the current path. If one is found, then it replaces the current one, and the algorithm continues until no shorter distance can be found just by changing two paths, and there is a 2-optimal tour.

At each iteration, the algorithm applies the best possible 2-opt move, if :

$$d(i,j) + d(i+1,j+1) - d(i,i+1) - d(j,j+1) < 0 \quad (3.16)$$

then replacing $(i,j), (i+1,j+1)$ with $(i,i+1), (j,j+1)$ minimizes the path length, Fig. 3.1 illustrates this.

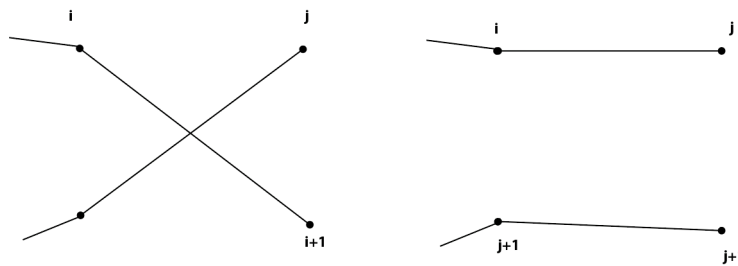


Figure 3.1: Visual example: 2-opt

The 2-opt, as well as the NNA, is intended for solving the TSP. The *problem* of the TSP concerns finding the shortest path connecting all the cities that a traveling salesman has to visit only once and then return to the starting point. In this case we are not interested in paths that are connected to the starting point (Main Substation) in a loop, therefore both the algorithms have been altered in order to have a fixed starting node and an open end.

The pairwise distances between all the nodes are collected in a distance matrix, *distmat* in (2). Since the distances must take into account the geography of the Nordhavn area it has been used

pathfinder [8], a Matlab code that provides a set of functions to identify the shortest path between two points with obstacle avoidance.

When the costs of the cables are introduced the main selection criteria is still the minimal cost, but this time it will rely on both the length and the cost for the single cable connection, hence the distance matrix is updated to *costmat* (3) so that the costs are taken into account in the process.

$$\text{distmat} = \begin{bmatrix} 0 & d(1,2) & d(1,3) & \dots & d(1,N) \\ d(2,1) & 0 & \ddots & \ddots & d(2,N) \\ d(3,1) & \ddots & \ddots & \ddots & \dots \\ \dots & \ddots & \ddots & \ddots & \dots \\ \dots & \ddots & \ddots & \ddots & d(N-1,N) \\ d(N,1) & d(N,2) & \dots & d(N-1,N) & 0 \end{bmatrix} \quad (2)$$

$$\text{costmat} = \begin{bmatrix} 0 & d(1,2) \cdot \text{Cost}(1,2) & \dots & d(1,N) \cdot \text{Cost}(1,N) \\ d(2,1) \cdot \text{Cost}(2,1) & \dots & \dots & d(2,N) \cdot \text{Cost}(2,N) \\ d(3,1) \cdot \text{Cost}(3,1) & \ddots & \ddots & \dots \\ \dots & \ddots & \ddots & \dots \\ \dots & \ddots & \ddots & \dots \\ d(N,1) \cdot \text{Cost}(N,1) & \dots & \dots & 0 \end{bmatrix} \quad (3)$$

For a better understanding an example of both the algorithms applied to a samples in Fig. 3.2 of nodes is shown in Figures 3.3 and 3.4. While the algorithm chooses the nearest neighbour, the total length of the grid is longer than the grid which is estimated by using the 2-opt based algorithm. The cost was set at 550000 Dkk per unit of length.

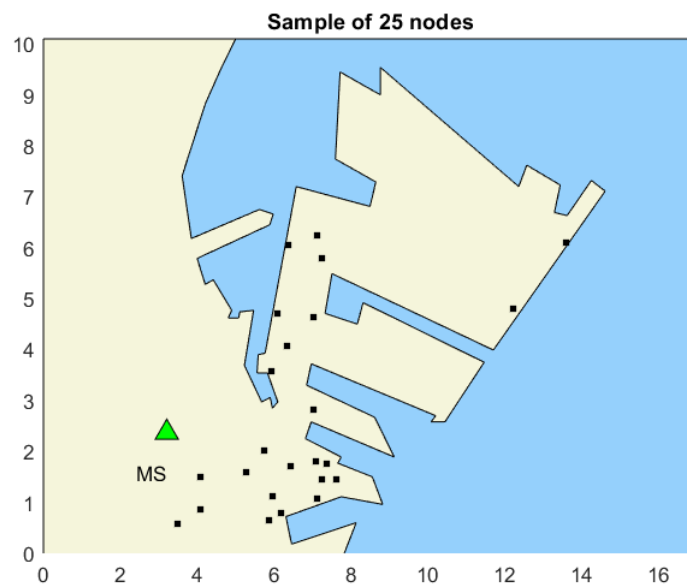


Figure 3.2: Visual example: sample of 25 nodes

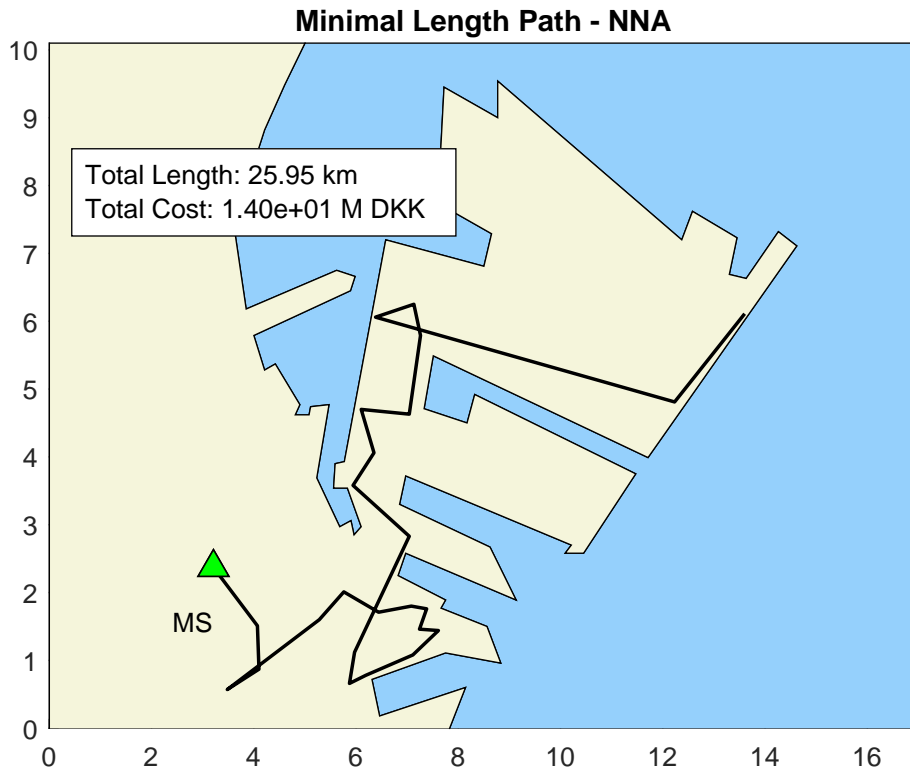


Figure 3.3: Visual example: NNA

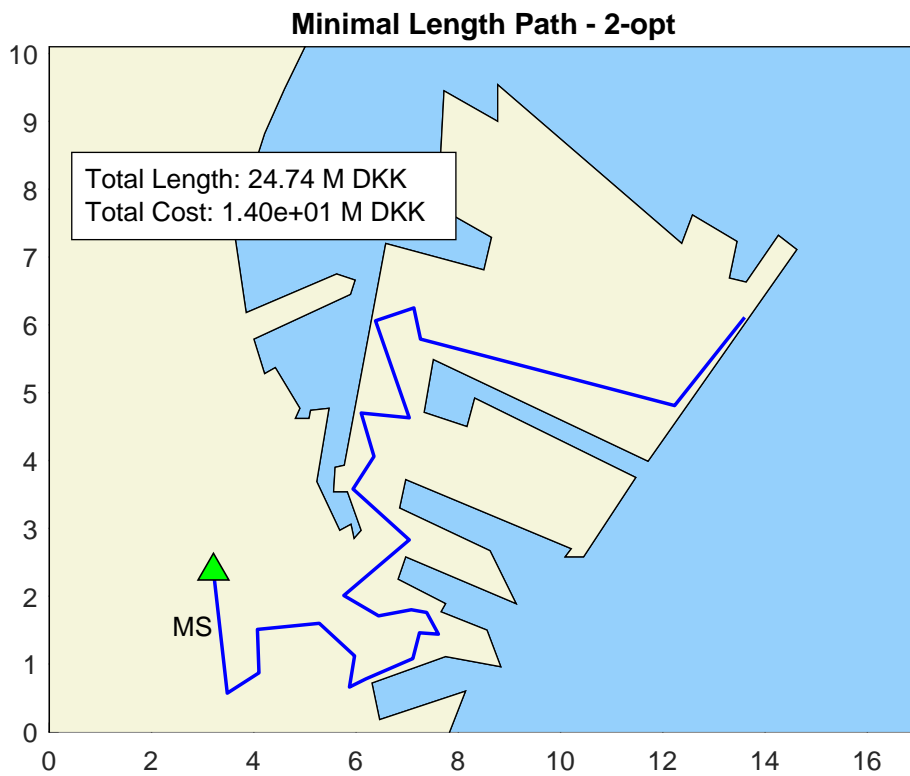


Figure 3.4: Visual example: 2-opt

3.4 3-Degree Vertex Constraint

Minimum spanning tree has direct application in the design of networks and it is used in algorithms approximating the TSP or similar problems. In its general formulation there is no constraint regarding the maximum degree of the graph.

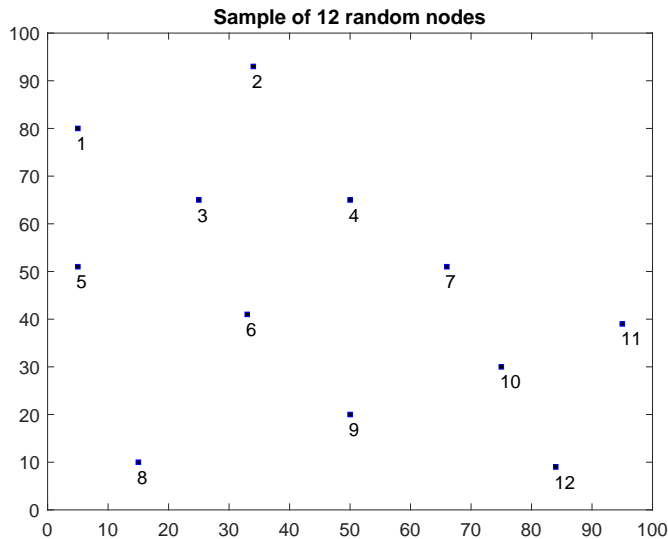
In our specific case 3-DCMST can be used to find a minimum spanning tree with 3-degree vertex constraint. During the execution of the algorithm each node V is either in the tree T or outside the tree.

$G=(V,E)$ is a graph with $V= [1, \dots, N]$ set of nodes representing different substations. In our case and $V=1$ representing the MS and E is a set of edges whose weights, distances or costs, are represented in a symmetric and no-negative matrix.

These are the steps of the algorithm:

1. Select a starting node V as origin;
2. While (there are still nodes outside the tree)
 - Find the edge of minimum weight between the unvisited node and the origin;
 - Add the selected edge and vertex to the tree;
 - Set the current node as new starting node, if $\text{deg}(V) = 3$ mark it as visited.

Examples shown in Fig. 3.6- Fig. 3.7



Index	X	Y
1	5	80
2	34	93
3	25	65
4	50	65
5	5	51
6	33	41
7	66	51
8	15	10
9	50	20
10	75	30
11	95	39
12	84	9

Figure 3.6: Sample of 12 random nodes

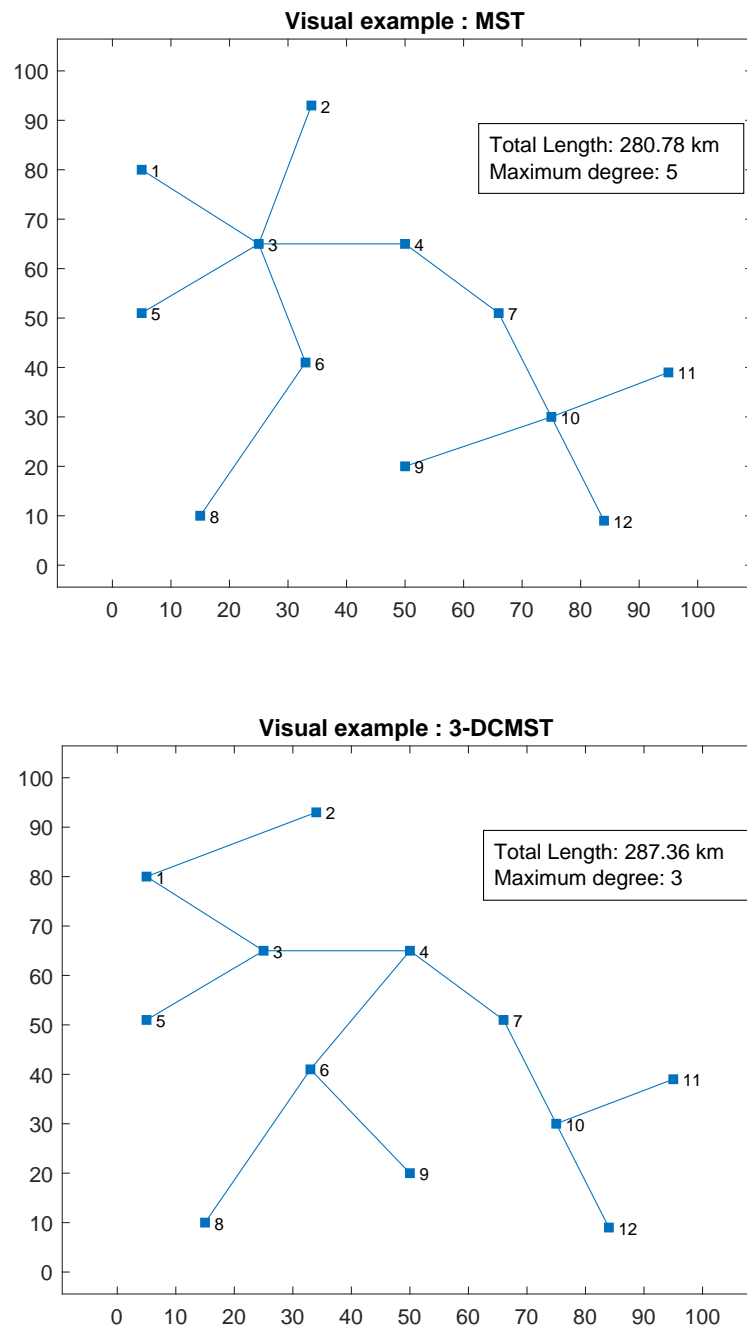


Figure 3.7: MST and 3-DCMST applied to a set of 12 nodes.

Chapter 4

Testing the Topology

Each topology will be tested through a power flow simulation using the program MATPOWER which is an open-source Matlab-language for solving steady-state power system simulation. MATPOWER analyse the system's capability to adequately supply the connected loads. The test is using the Indexes of the substations according to their configuration in the grid, the data consumption and the type of cable used for each connection. The data power consumption, as said before, is the hourly consumption. In this time scenario the peaks occur all at the same time.

4.1 Case 1 : Minimal Length - 2 Edges

The aim is to find the Minimal Length Path connecting all the substations using a single route, the individual cables connecting the single substations are considered equal, therefore just one cable type is used and the total cost depends only on the total length of the path. The topology is shown in Fig 4.1. C_1 has been used as fixed cable type shown. The results related to a different choice of cable type will be presented in the next section.

4.2 Case 2 : Minimal Length + Cable Size - 2 Edges

As expected, using different selection criteria, the total cost is increasing as more constraints are introduced. However, the Minimal Length is still the determining factor, thus the total distance might be the same or slightly different for both single and multiple cable category.

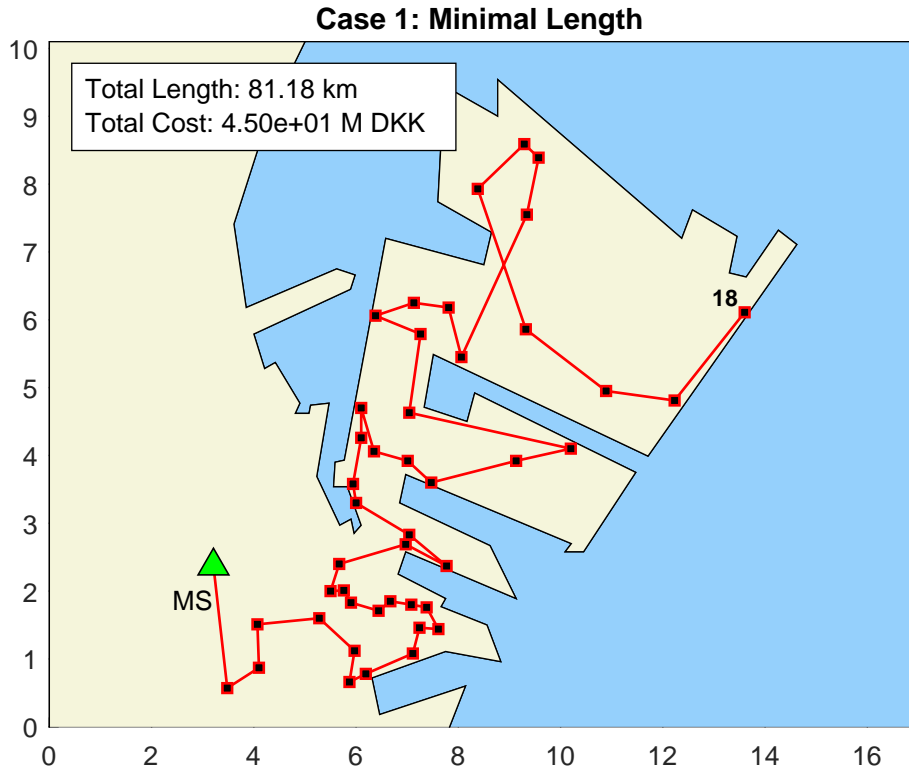
The outputs of the topologies generated in Case 1 and in Case 2 are presented in Table 4.1

	Case 1 (C1)	Case 1 (C2)	Case 1 (C3)	Case 1 (C4)	Case 2
TotalCost (M Dkk)	44	58	67	71	8.63
TotalDistance (km)	81.18	81.18	81.18	81.18	82.27
TotalReliability (%)	0.28	0.28	0.28	0.28	0.27

Table 4.1: Comparison between Case 1 and Case 2

The results met our expectations:

$$\text{TotalCost}_{\text{Case1}} < \text{TotalCost}_{\text{Case2}}$$



$$\text{TotalDistance}_{\text{Case1}} < \text{TotalDistance}_{\text{Case2}}$$

both total distance and total cost of the topology increased, but if we consider the total reliability index:

$$\text{rRI}_{\text{Case1}} \approx \text{rRI}_{\text{Case2}} < \text{minRI}$$

differ only slightly, and in both cases is lower than the minimum Reliability Index accepted. The probability of failure is actually higher than the probability of performing successfully, this means that both the topologies are not acceptable.

To confirm this analysis we can run a power flow simulation using MATPOWER.

If the Voltage Magnitude is in the range 0.9 – 1.1 p.u., (110% – 90%) at each bus it is possible to say that the grid obtained with the algorithm is stable. For every stable grid it is worth analyzing the losses, that are not considered in the objective function of the algorithm proposed, but it is necessary to take into consideration for future works. In both Case 1 and Case 2 the minimum Voltage Magnitude is respectively 0.723 p.u. and 0.662 p.u. both at bus 18777. The Index 18, labeled in Figures 4.1 and 4.2, represent the last substation connected to the grid at 10 kV level while the three consecutive 7 in 18777 indicate that the minimum occurs at 0.4 kV level. The complete table can be found in Appendix B.

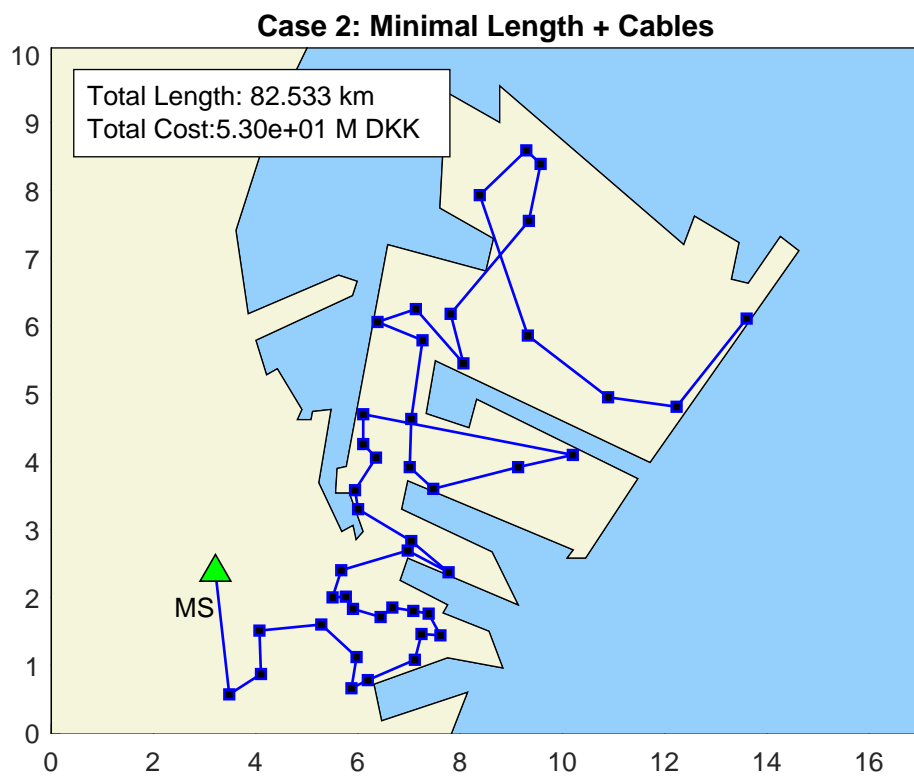


Figure 4.2: Minimal Length + Cable Size

Voltage Test - Case 1

1 Newton's method power flow converged in 4 iterations.

2 Converged in 0.22 seconds

3

4 | System Summary

5

6 How many?		7 How much?	P (MW)	Q (MVar)
8 Buses	89	Total Gen Capacity	20.0	-20.0 to 20.0
9 Generators	1	On-line Capacity	20.0	-20.0 to 20.0
10 Committed Gens	1	Generation (actual)	6.8	2.1
11 Loads	41	Load	5.6	1.8
12 Fixed	41	Fixed	5.6	1.8
13 Dispatchable	0	Dispatchable	-0.0 of -0.0	-0.0
14 Shunts	0	Shunt (inj)	-0.0	0.0
15 Branches	88	Losses ($I^2 * Z$)	1.20	0.50
16 Transformers	44	Branch Charging (inj)	-	0.2
17 Inter-ties	0	Total Inter-tie Flow	0.0	0.0
18 Areas	1			

19

20

Minimum

Maximum

21

22 Voltage Magnitude	0.723 p.u. @ bus 18777	1.000 p.u. @ bus 1
23 Voltage Angle	-151.99 deg @ bus 18777	0.00 deg @ bus 1
24 P Losses ($I^2 * R$)	-	0.09 MW @ line 1-23
25 Q Losses ($I^2 * X$)	-	0.04 MVar @ line 1-23

Voltage Test - Case 2

1 Newton's method power flow converged in 5 iterations.

2

3 Converged in 0.05 seconds

4

5 | System Summary

6

7 How many?		8 How much?	P (MW)	Q (MVAr)
9 Buses	89	Total Gen Capacity	20.0	-20.0 to 20.0
10 Generators	1	On-line Capacity	20.0	-20.0 to 20.0
11 Committed Gens	1	Generation (actual)	7.0	2.2
12 Loads	41	Load	5.6	1.8
13 Fixed	41	Fixed	5.6	1.8
14 Dispatchable	0	Dispatchable	-0.0 of -0.0	-0.0
15 Shunts	0	Shunt (inj)	-0.0	0.0
16 Branches	88	Losses ($I^2 * Z$)	1.45	0.62
17 Transformers	44	Branch Charging (inj)	-	0.2
18 Inter-ties	0	Total Inter-tie Flow	0.0	0.0
19 Areas	1			

20

21

22

	Minimum	Maximum
23 Voltage Magnitude	0.662 p.u. @ bus 18777	1.000 p.u. @ bus 1
24 Voltage Angle	-152.54 deg @ bus 18777	0.00 deg @ bus 1
25 P Losses ($I^2 * R$)	-	0.09 MW @ line 1-23
26 Q Losses ($I^2 * X$)	-	0.04 MVAr @ line 1-23

4.3 Case 3 - Minimal Length + Cable Size + Reliability - 2 Edges

As discussed in the previous sections the Reliability Index is a key factor in the process of selecting the optimal topology. In Case 1 and Case 2 the rRI and tRI have been evaluated but no actions have been taken to improve them. Thus the outcomes of the the first attempts are rejected. In this section the algorithm is going to operate on an existing topology (Case 2) and specifically checking whether or not the tRI is acceptable. If not, the topology network, which configuration consists of only one *path* exiting the MS, most likely will be split in more branches. The resulting topology can be seen in Fig 4.3

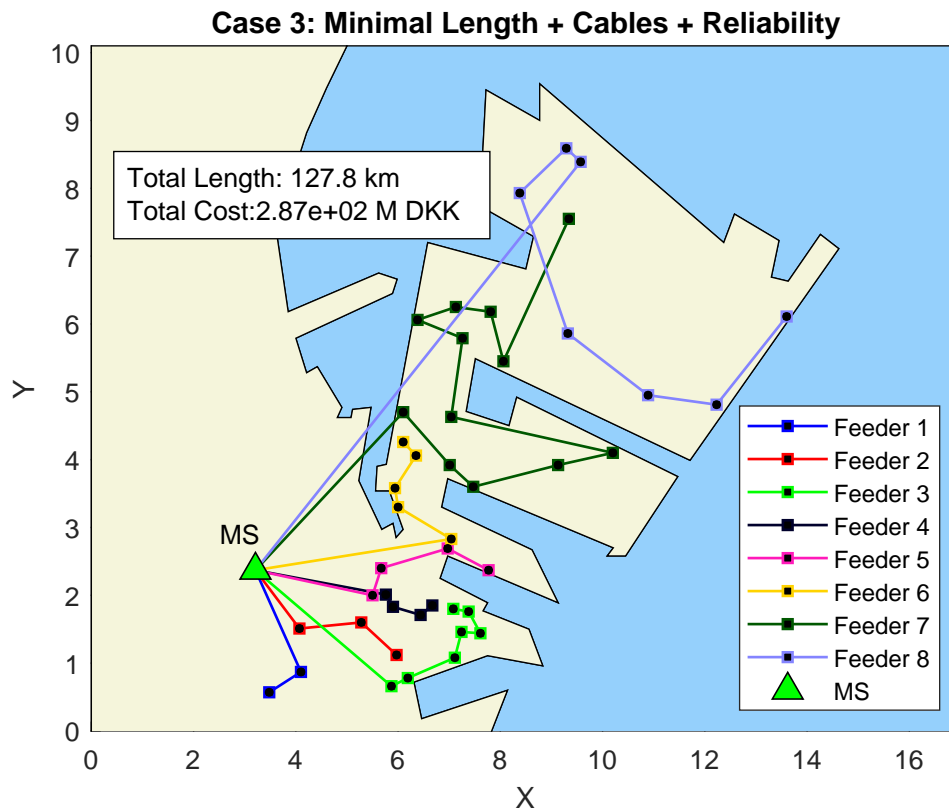


Figure 4.3: Case 3

By comparing the outcome of Case 3 to the previous two, we can see that the algorithm acted on the rRI to improve the overall Reliability of the network. For this purpose the initial topology has been split in 6 branches. The related results are shown in Fig. 4.3, where the $tRI_{Case\ 3}$ value is referred to the minimum tRI value that occurs in Feeder 1.

Testing Case 3

	Minimum	Maximum
1		
2		
3 Voltage Magnitude	0.970 p.u. @ bus 30777	1.000 p.u. @ bus 1
4 Voltage Angle	-150.26 deg @ bus 30777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.01 MW @ line 1-45
6 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 1-45

As we can see in Case 3 the minimum Voltage Magnitude is 0.970 p.u., which is between the [0.9, 1.1] range, therefore acceptable and occurs at 30777, at 0.4 kV level. The maximum Voltage Angle is 0, the minimum voltage angle is -150.51 , that is included in the acceptable range. This grid is stable and can be considered a successful result.

4.4 Case 4 - Minimal Length + Cable Size + Reliability + N-1 - 2 Edges

The N-1 secure upgrade is intended to improve the overall Reliability of the network. In fact in case of failure of any of the cables, a reserve cable is provided in order to supply all the substations affected by the failure restoring the connection, that has been interrupted to the MS and therefore providing electric power again. The Reserve Cables are added to the topology using the Minimal Length Criteria: the free edges (the one connected to one edge only, excluding the MS) of the branches are connected with the shortest path possible, thus providing the least expensive solution.

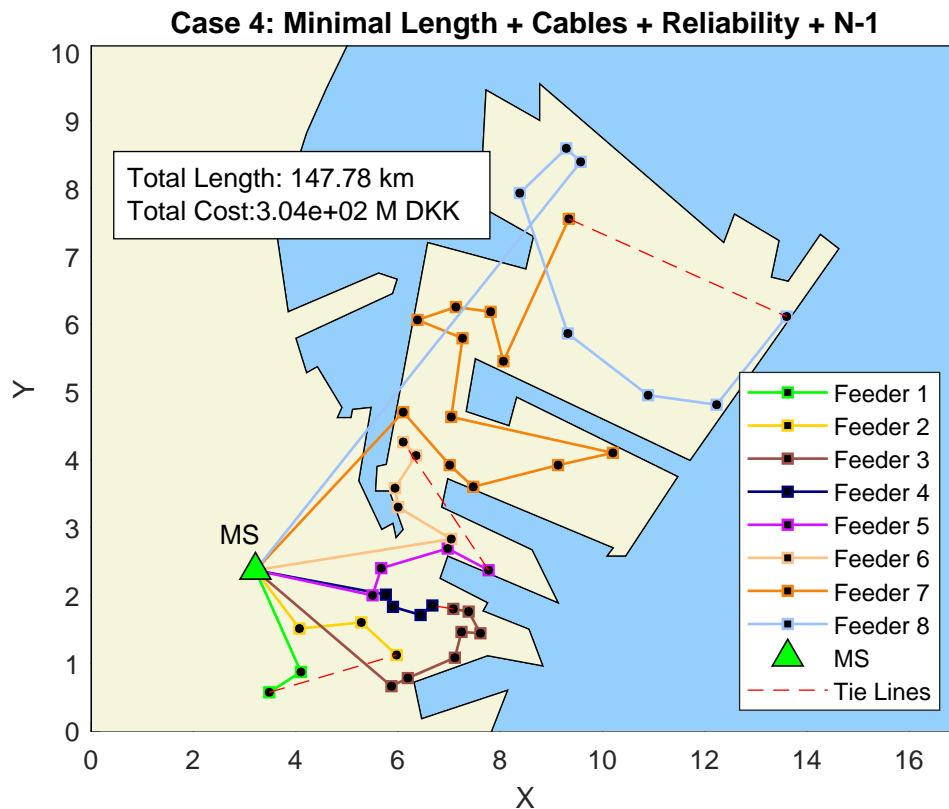


Figure 4.4

While this solution is improving the total Reliability of the system, the total cost will increase since the power grid is larger than in Case 3 and therefore it will be more expensive in the construction phase. Additionally, the reserve cables have to carry a total load higher than usual when in use, thus necessitating cables of bigger size. As we can see in Case 4 the minimum Voltage Magnitude is 0.990 p.u. acceptable and occurs at 20777, at 0.4 kV level. The maximum Voltage Angle is 0, the minimum voltage angle is -150.10, which is within the acceptable range. As for Case 3 this grid is stable and can be considered a successful result.

Testing Case 4

	Minimum	Maximum
1		
2		
3	Voltage Magnitude 0.990 p.u. @ bus 20777	1.000 p.u. @ bus 1
4	Voltage Angle -150.10 deg @ bus 20777	0.00 deg @ bus 1
5	P Losses ($I^2 \cdot R$) -	0.00 MW @ line 1-39
6	Q Losses ($I^2 \cdot X$) -	0.00 MVar @ line 1-45

Index	Sub1	Sub2	X1	Y1	X2	Y2	Peaks	Distance (km)	Cables	Cost (M DKK)
45	33	43	6.67	1.85	7.77	2.37	264.2	0.5	4	0.5
46	15	44	6.44	1.71	7.01	3.92	428.75	3.897	4	3.4
47	9	30	5.97	1.12	9.4	7.55	170.78	5.1	4	4.5
48	23	18	3.48	0.57	13.6	6.11	409.02	8.9936	4	8

Table 4.2: Case 5 : Reserve Cable Specifics

4.5 Summary

Using the formulas from Equations (2.13) - (2.15), for Case 3 and 4:

$$\text{SAIFI} = 0.14 \text{ t/y}$$

$$\text{SAIDI}_1 = 2.22 \text{ h/y}$$

$$\text{SAIDI}_2 = 0.58 \text{ h/y} \quad (4.1)$$

SAIDI in general represents the System Average Interruption Duration Index, per year. The value SAIDI₂ refers to average interruption duration with a N-1 secure system, Case 4, and as expected its value is close to 0.5 h/y. SAIDI₁ refers to the topology designed in Case 3 which is not equipped with the reserve cables. As expected the results show that the estimated duration of interruption of electrical power per year is lower if the power grid system is N-1 contingent since the ARST is in average shorter than the ART. SAIFI is the same.

Table 4.3 compare the total cost and the total distance for 4 Cases.

2 - Edges				
	Case 1 (C1)	Case 2	Case 3	Case 4
TotalCost (M Dkk)	44	53	287	304
TotalDistance (km)	81.2	82.5	128	147.8

Table 4.3: Comparing the 4 Cases

4.6 Case 1 to Case 4 - 3 Edges

The aim is to find the design a topology for each Case as done in Sections 4.1 to 4.4.

This time the nodes are connected at most to three edges,

$$\forall i \quad \text{deg}(i) \leq 3$$

as explained in Section 3.4.

Case 1 and Case 2

The designed topologies are shown in Figures 4.5 and 4.6.

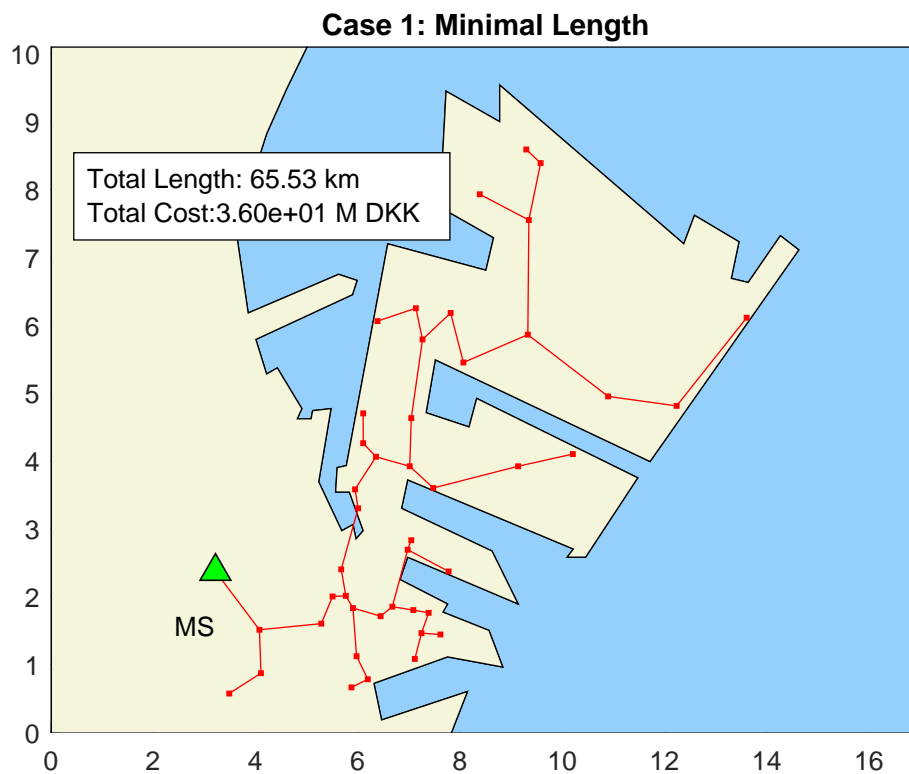
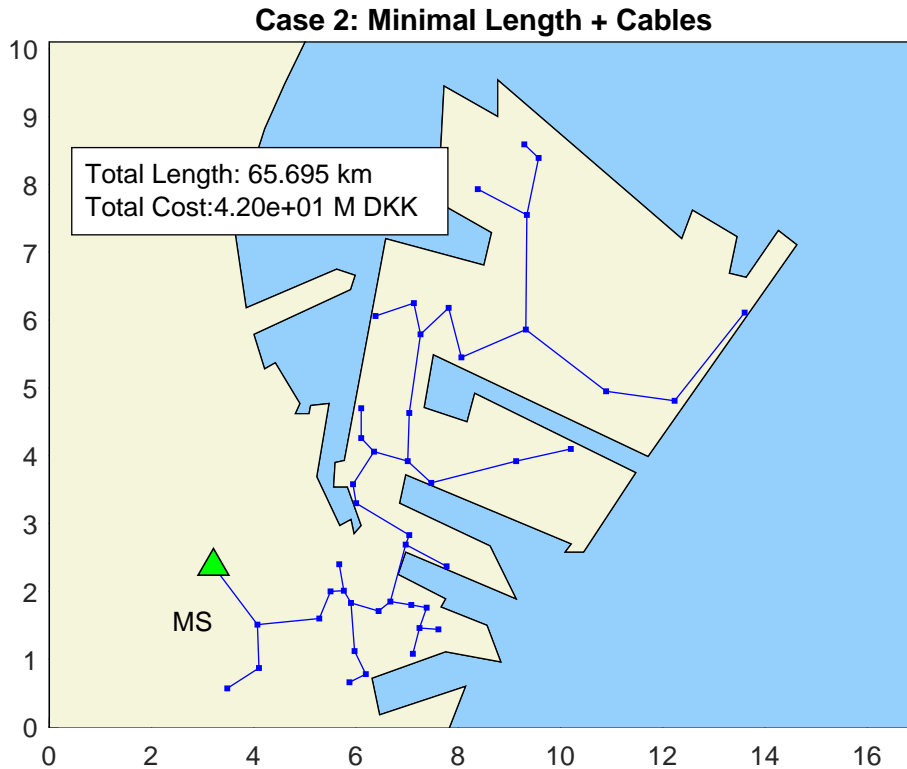


Figure 4.5: Case 1 : 3 Edges

Voltage Test - Case 1 - 3Edges

1	Minimum	Maximum
2		
3 Voltage Magnitude	0.855 p.u. @ bus 37777	1.000 p.u. @ bus 1
4 Voltage Angle	-150.92 deg @ bus 37777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.11 MW @ line 1-22
6 Q Losses ($I^2 \cdot X$)	-	0.05 MVar @ line 1-22



Voltage Test - Case 2 - 3Edges

1	Minimum	Maximum
2		
3 Voltage Magnitude	0.751 p.u. @ bus 18777	1.000 p.u. @ bus 1
4 Voltage Angle	-151.70 deg @ bus 18777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.16 MW @ line 14-44
6 Q Losses ($I^2 \cdot X$)	-	0.08 MVar @ line 14-44

The outcomes of Case 1 and Case 2 with 3-Edges constraint don't offer a valid topology. The results differ only slightly from the 2-Edges solutions showed in 4.1 and 4.2. The total reliability (4.4) is again too low and looking at the values of the voltage test the minimum Voltage Magnitude is outside the [0.9, 1.1] range for both the designed grids.

For the given data set a topology that involve a single line connected all the substations is not feasible.

	Case 1 (C1)	Case 2
TotalCost (M Dkk)	36	42
TotalDistance (km)	63.53	65.7
TotalReliability (%)	0.33	0.33

Table 4.4: Comparing Case 1 and Case 2 (3-Edges)

Case 3 and Case 4

The designed topologies are shown in Figure 4.7 and 4.8 .

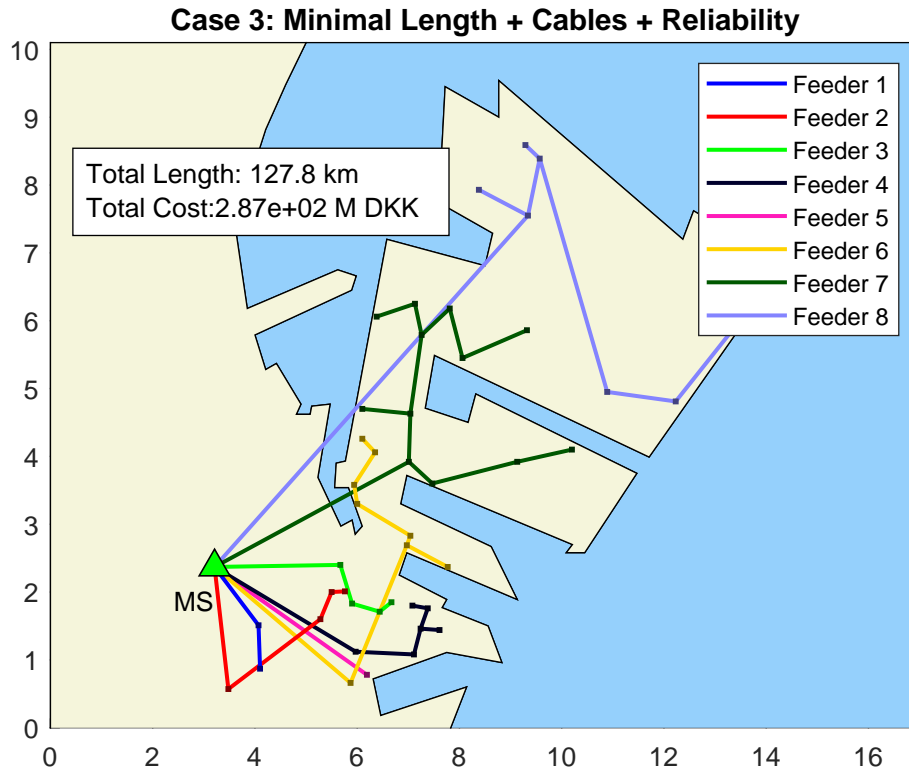


Figure 4.7: Case 3 : 3 Edges

Minimum Maximum

Voltage Test - Case 3 - 3Edges

1	Voltage Magnitude	0.978 p.u. @ bus 40777	1.000 p.u. @ bus 1
2	Voltage Angle	-150.19 deg @ bus 40777	0.00 deg @ bus 1
3	P Losses ($I^2 \cdot R$)	-	0.03 MW @ line 1-44
4	Q Losses ($I^2 \cdot X$)	-	0.01 MVar @ line 1-44

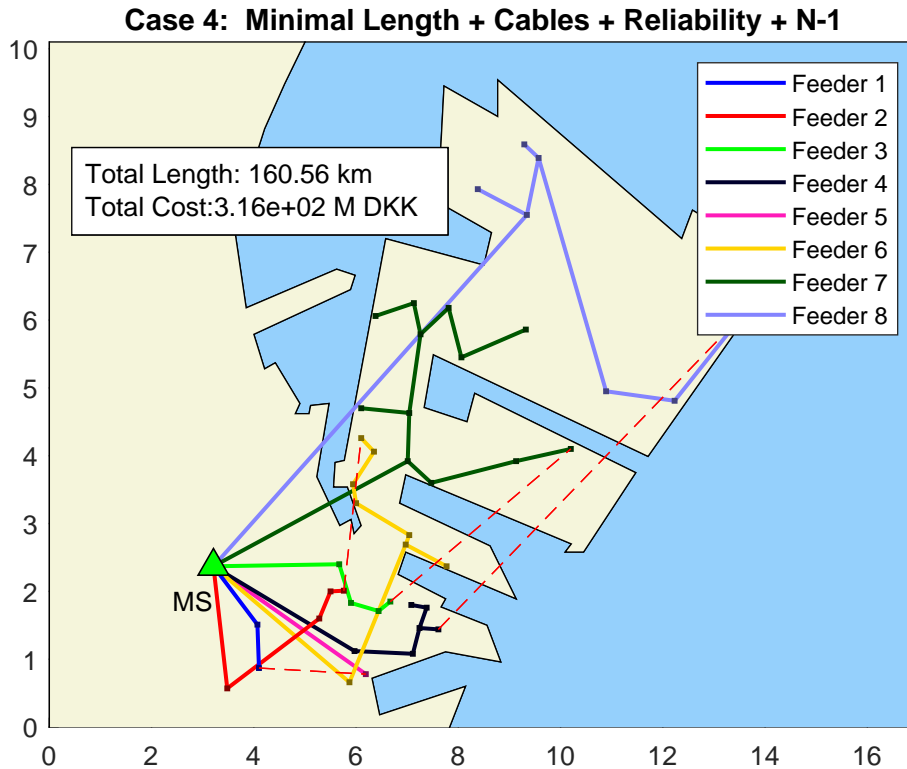


Figure 4.8: Case 4 : 3 Edges

Voltage Test - Case 4 - 3Edges

	Minimum	Maximum
1		
2		
3 Voltage Magnitude	0.972 p.u. @ bus 30777	1.000 p.u. @ bus 1
4 Voltage Angle	-150.22 deg @ bus 30777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.01 MW @ line 27-5
6 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 27-5

4.7 Summary

Using the formulas from Equations (2.13) - (2.15), for Case 3 and 4:

$$\begin{aligned}
 \text{SAIFI} &= 0.16 \text{ t/y} \\
 \text{SAIDI}_1 &= 2.55 \text{ h/y} \\
 \text{SAIDI}_2 &= 0.64 \text{ h/y}
 \end{aligned}
 \tag{4.2}$$

As for the results found for the 2-Edges topologies, the values of SAIFI, SAIDI₁ and SAIDI₂ are close to the annual averages [5]. At a first comparison it is notable that the values are overall higher than in 4.1 and that SAIDI₂ < SAIDI₁ again.

3 - Edges				
	Case 1 (C1)	Case 2	Case 3	Case 4
TotalCost (M Dkk)	36	42	287	316
TotalDistance (km)	63.53	65.7	127	160

Table 4.5: Comparing the 4 Cases

Table [?] shows that the total length and the total cost of the 3-Edges topology found in Case 4 are higher compared to the 2-Edges in Table [?], while the results for Case 1 and Case 2 would suggest a better outcome. This might suggest that the algorithm is implemented inefficiently. This result might be related to the criteria used for positioning the reserve cables, which are added to the topology using the Minimal Length Criteria: the free edges, the ones connected to one edge only, MS excluded, are connected with the shortest path possible in order to provide the least expensive solution. This method can provide an optimal solution for the 2-Edges, but it must be implemented in order to guarantee a better result for the 3-Edges case that might present more free edges. Thus it might require either a different selection criteria or more reserve cables in order to guarantee power to all the substation in case of need.

Chapter 5

Chapter 4

5.1 Case 5 : Time Periods

In Chapter 3 the optimization solver has been tested using the assumption that the peaks of consumption for all substations occur at the same time, that represents the worst case scenario.

The peaks of consumption used in this Chapter are from different time periods, namely:

- Christmas : 24th and 25th of December.
- Summer Vacation : from the 30th of June to 14th of July .
- New Year's Eve : 31th of December.
- January : from the 8th to the 17th of January.

All the data refers to the year 2006 and represent the hour power consumption per substation. The choice of the time scenarios was arbitrary, but not random.

The peak demand of a system is the highest demand that has occurred over a specified time period. The peak can depend on the weather, the climate, even the day of the week or a specific hour, as well as other factors. For example in industrialized areas the peak demands would mostly occur during the day time. Seasonal factors, other than climate-related, can induce more people to stay at home and make heavier or extended use of electric appliances. These periods include vacations, Christmas season and holidays, in general. The peak demands are most likely to occur in different day of the week or time of the day for different categories of customers.

While Christmas, Summer and New Year's Eve are holidays periods, the window of time selected in January is mostly working days and usually the coldest time of the year. January has been chosen because presents the largest occurrence of peaks, over the all year.

The aim is to design a topology for each data set including all the constrains as done in Section 4.4 for Case 4. And overview of the designed 2-Edges topologies is shown in the following sections. Detailed tables and plots for the 2-Edges and the 3-Edges networks are presented in B and A. For matter of simplicity the topologies provided for each time period are named respectively::

Peaks occur at the same time (Peak Period): Topology P.

Christmas : Topology C.

Summer Vacation : Topology S .

New Year's Eve : Topology N.

January : Topology J.

5.2 Case 4 : Christmas

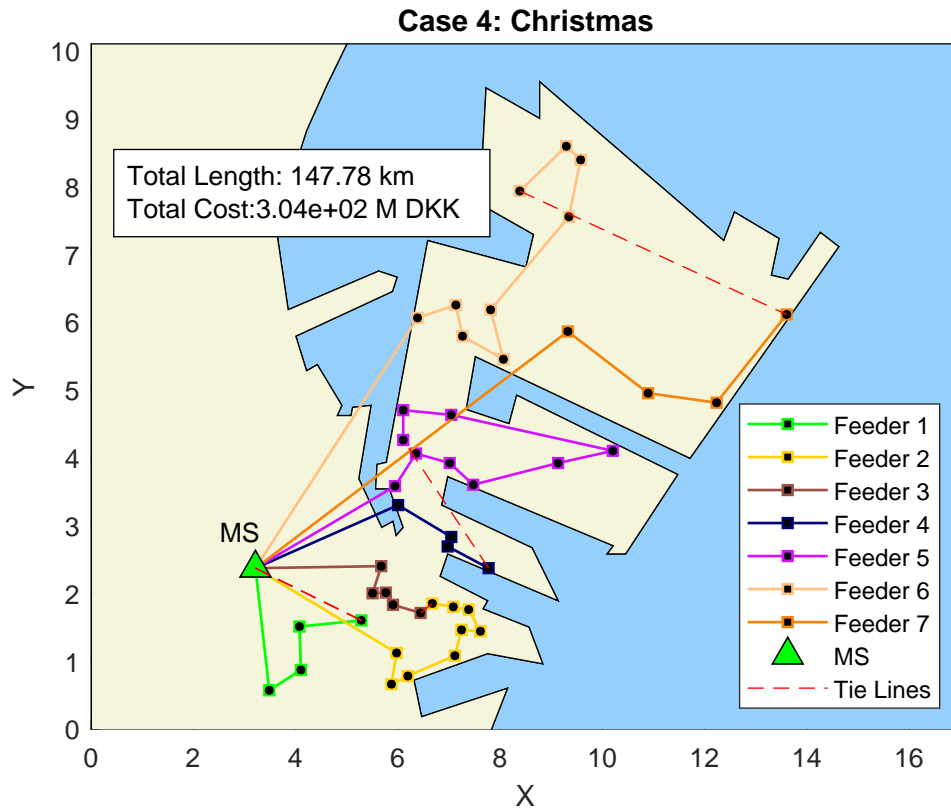


Figure 5.1: Topology C

Voltage Test - Christmas

	Minimum	Maximum
1		
2		
3 Voltage Magnitude	0.994 p.u. @ bus 41777	1.000 p.u. @ bus 1
4 Voltage Angle	-150.07 deg @ bus 41777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.00 MW @ line 1-11
6 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 1-11

5.3 Case 4 : Summer Vacation

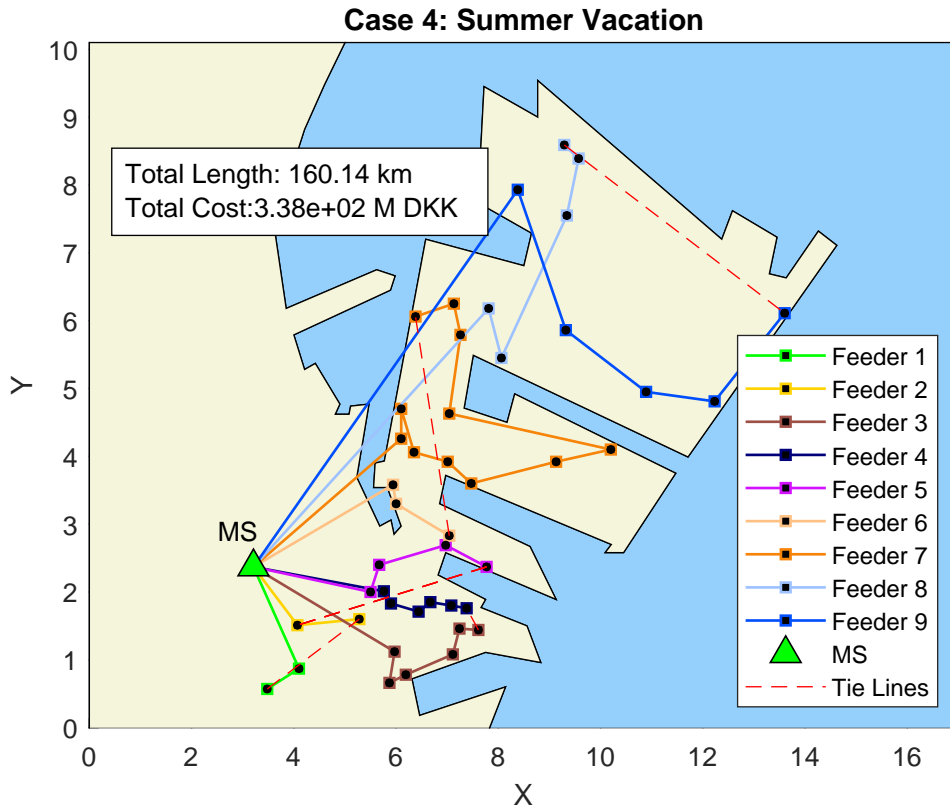


Figure 5.2: Topology S

Voltage Test - Summer

	Minimum	Maximum
1		
2		
3		
4 Voltage Magnitude	0.984 p.u. @ bus 20777	1.000 p.u. @ bus 1
5 Voltage Angle	-150.14 deg @ bus 20777	0.00 deg @ bus 1
6 P Losses ($I^2 \cdot R$)	-	0.00 MW @ line 38-40
7 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 38-40

5.4 Case 4 : New Year's Eve

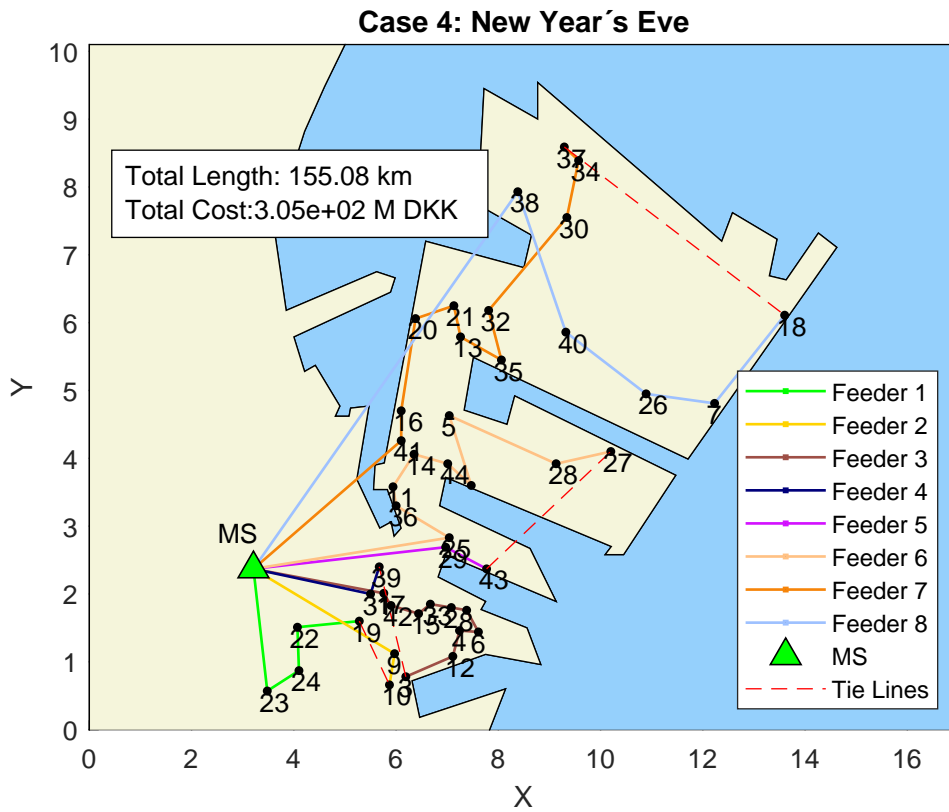


Figure 5.3: Topology N

Voltage Test - New Year's Eve

	Minimum	Maximum
1		
2		
3 Voltage Magnitude	0.989 p.u. @ bus 37777	1.000 p.u. @ bus 1
4 Voltage Angle	-150.09 deg @ bus 37777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.00 MW @ line 1-25
6 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 1-25

5.5 Case 4 : January

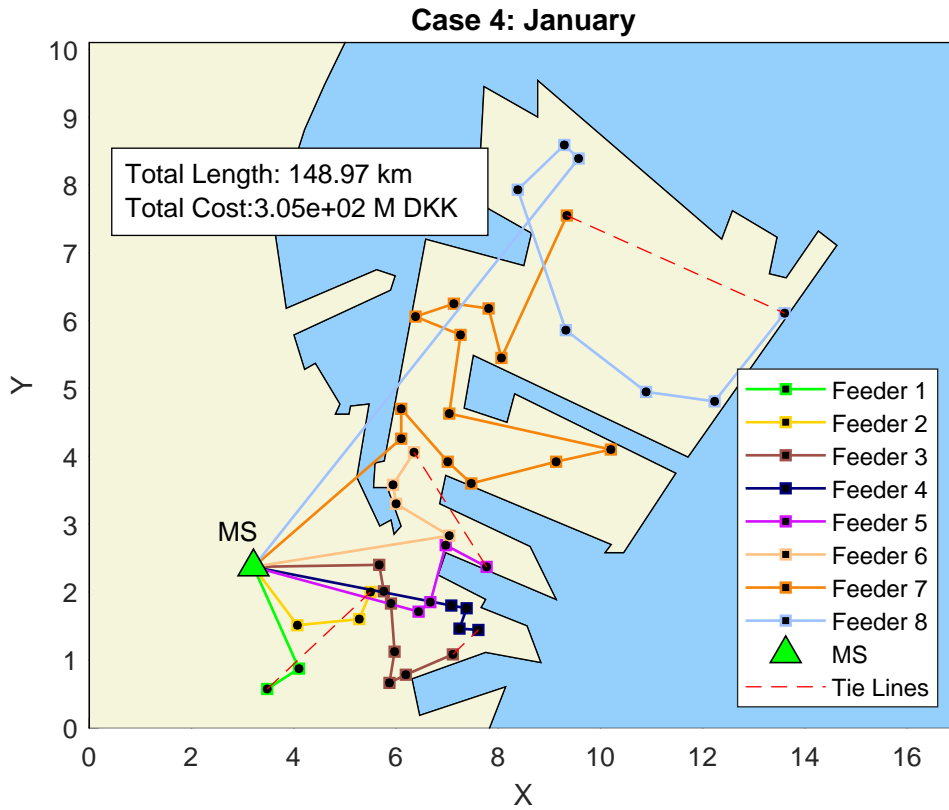


Figure 5.4: Topology J

Voltage Test - January

	Minimum	Maximum
1		
2		
3 Voltage Magnitude	0.960 p.u. @ bus 30777	1.000 p.u. @ bus 1
4 Voltage Angle	-150.31 deg @ bus 30777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.01 MW @ line 44-45
6 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 37-38

The voltage tests show that all the topologies designed by the optimization tool have minimum Voltage Magnitude between $[0.9, 1.1]$, with a range from the minimum value of 0.96 occurring at New Year's Eve to a maximum of 0.988 p.u. occurring during Christmas. SAIFI and SAIDI values don't differ much from each others. A summary of the results is presented in Table 5.1, where the four designed grids are compared to Topology P found in Section 4.4, Case 4, with the assumption that all the peaks occur at the same time. Overall the differences in terms of reliability and voltage analysis are not significant.

It is important to mention that in the above tables the voltage has been tested exclusively using the peaks of demand used as input of the optimization solver, meaning that the results can be different running the voltage test for Topology P using the peak demand from Christmas:

Voltage Test: Topology P - Christmas

1	Minimum	Maximum
2	-----	-----
3	-----	-----
4 Voltage Magnitude	0.991 p.u. @ bus 30777	1.000 p.u. @ bus 1
5 Voltage Angle	-150.14 deg @ bus 30777	0.00 deg @ bus 1
6 P Losses ($I^2 \cdot R$)	-	0.00 MW @ line 1-16
7 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 1-16

Running the voltage test for Topology C using the peak demand from Christmas:

Voltage Test: Topology C - Peak Period

1	Minimum	Maximum
2	-----	-----
3	-----	-----
3 Voltage Magnitude	0.980 p.u. @ bus 38777	1.000 p.u. @ bus 1
4 Voltage Angle	-150.13 deg @ bus 38777	0.00 deg @ bus 1
5 P Losses ($I^2 \cdot R$)	-	0.01 MW @ line 1-20
6 Q Losses ($I^2 \cdot X$)	-	0.00 MVar @ line 1-20

Case 4: Different Time Scenarios					
	Peak Period	Christmas	Summer	New Year's Eve	January
Total Length (km)	160	147.8	160	155	149
Total Cost (M Dkk)	316	304	322	305	305
SAIFI	0.14	0.14	0.14	0.12	0.14
SAIDI ₁	2.3	2.2	2.3	2.0	2.2
SAIDI ₂	0.58	0.55	0.57	0.49	0.54

Table 5.1: Case 5 : summary of the results

Chapter 6

Conclusion

This project has shown one way in which mathematical optimization can be used in order to analyze the efficiency and the reliability of a power grid. It is necessary to implement both optimization tools themselves and the analysis methods in order to achieve a complete quantitative evaluation of the designed topology.

This report first stated the criteria and the constraints used as foundation of the problem of designing an optimal electrical power network, describing the goals and the expected outcomes.

Given the new configuration of the substations the optimization process started solving the Minimal Length problem Case 1 and proceeded adding one by one a new constraint in the designing process, namely Cables in Case 2, Reliability in Case 3 and achieving a N-1 secure topology in Case 4. Looking at the results it is notable that:

$$\begin{aligned} \text{TotalDistance}_{\text{Case}_1} &< \text{TotalDistance}_{\text{Case}_2} < \text{TotalDistance}_{\text{Case}_3} < \text{TotalDistance}_{\text{Case}_4} \\ \text{TotalCost}_{\text{Case}_1} &< \text{TotalCost}_{\text{Case}_2} < \text{TotalCost}_{\text{Case}_3} < \text{TotalCost}_{\text{Case}_4} \end{aligned}$$

while the reliability:

$$\text{Reliability}_{\text{Case}_1} \approx \text{Reliability}_{\text{Case}_2} < \text{Reliability}_{\text{Case}_3} = \text{Reliability}_{\text{Case}_4} \tag{6.1}$$

The selection of the optimal topology has to take into account different criteria. The final cost of the power grid can't be the only key factor in the decision process. A larger investment for a N-1 secure power network can provide a better service in terms of reliability. The topologies modelled in Case 1 - 2, as expected, don't provide a feasible network design for any of the time scenarios they have been tested with, since both the reliability and the minimum Voltage Magnitude are outside the accepted ranges of respectively respectively $[0.9, 1]$ and $[0.9, 1.1]$. This confirms that, at least regarding the set of nodes that has been tested, it's not possible to obtain a single line solution that is optimized in terms of minimal length while still fulfilling the reliability criteria.

On the other hand Case 3 - 4 always showed valid solutions and their comparison suggests the an N-1 secure network, even if requires a larger investment in the construction phase, can guarantee an higher reliability of the system in terms of SAIDI.

The topologies resulting from Chapter 4 and 5 are all feasible. According to the results presented in Sections 4.4 and 5.1 designing a power distribution network considering high peak demand can provide a well performing power grid as characterized by a high Reliability. Running the power flow test using the peak demand data from four different time scenarios, the performance of the network is improving in term of power flow test.

6.1 Future work

Additional criteria and parameters should be taken into account in order to provide a designed solution as much accurate as possible. In terms of expenses additional costs can be introduced such as penalty for violating reliability criteria or the cost of resupplying point for each feeder in case of outages as well as the cost of the electrical losses, that are inevitable.[5]

The introduction of the electrical losses can also given another parameter of comparison between different designs.

While evaluating the reliability of the system only the distribution lines, which move power from one location to another, were considered, specifically the failure rate associated to the typology of cable used and its length, although transformers, that change the voltage level of the electrical power, were not taken into account.[5] Additional components that might be included in the analysis of the designed grid is the reliability of the system might be protective equipment, which provides safety and "fail safe" operation and voltage regulation equipment, which is used to maintain voltage within an acceptable range as the load changes.[5] In this case the failure rate would be the sum of the failures rates associated to the specific component.

The optimization solver developed in this project is based on solving a constrained routing problem. Given the set of substations, the first approach has been to solve the TSP to find a primary topology connecting all the nodes in unique line to alter later on in the optimization process. Since the literature about routing optimization problems is pretty wide, other approaches can be taken into consideration. One option is to use the *cluster analysis*: grouping a set of objects in such a way that objects in the same group, called a *cluster*, are similar to each others. The objects in the analysis are the nodes that can be grouped by their distance or using another characteristic, the *clusters* could be then used as to generate an initial topology.

The optimization solver can be improved; for Case 4, N-1 secure, the positioning of the reserve cables is selected using as only constraint the total cost, therefore once chosen the cable type the shortest cable connection is selected, while more selection criteria should be considered and added in the analysis. The 3-Edges optimization solver can be improved and implemented in order to guarantee an optimal solution while designing a N-1 secure network.

Using either real distances or space coordinates would be beneficial in order to design a topology that can be potentially applied in reality. It would also make it easier to compare the "optimal" grid to the existing network, and to actually test the efficiency of the optimization tool developed.

Bibliography

- [1]
- [2] From the grid to the smart grid, topologically.
- [3] Minimum spanning trees.
- [4] Optimal planning of urban distribution network considering its topologys.
- [5] Economy profile denmark - doing business, 2018.
- [6] North Carolina U.S.A. H. Lee Willis ABB, Inc. Raleigh. *Power Distribution Planning Reference Book*. 204. [MARCEL DEKKER].
- [7] Roger A. Freeman Hugh D. Young. *Sears and Zemansky's University Phisiscs with Modern Physics Technology Update*. 13th edition.
- [8] Mohd Faiz Abd Razak. Shortest path identification with obstacle avoidance, 2011. [Online; Updated 05 Oct 2011].
- [9] Ronald L. Rivest Clifford Stein Thomas H. Cormen, Charles E. Leiserson. *Introduction to Algorithms*. 3rd edition.

Appendices

Appendix A

Tables

Links	Sub1	Sub2	X1	Y1	X2	Y2	Cables	Cost (M Dkk)	Pks (kW)	Dist (km)
1	1	2	3.2	2.4	3.5	0.57	1	2.2	111	3.9
2	2	3	3.5	0.57	4.1	0.87	1	0.74	100	1.4
3	3	4	4.1	0.87	4.5	0.94	1	0.39	58	0.7
4	4	5	4.5	0.94	5.4	1.4	1	1.3	300	2.3
5	5	6	5.4	1.4	5.7	1	1	0.64	65	1.2
6	6	7	5.7	1	6	1.3	2	1.2	97	1.7
7	7	8	6	1.3	5.9	0.66	2	2.4	85	3.3
8	8	9	5.9	0.66	6.2	0.78	2	2.5	18	3.5
9	9	10	6.2	0.78	6.5	0.78	1	5.1	266	9.2
10	10	11	6.5	0.78	6.8	0.89	1	5.2	045	9.4
11	11	12	6.8	0.89	7.1	1.1	1	0.53	34	0.96
12	12	13	7.1	1.1	6.4	1.7	1	0.85	144	1.5
13	13	14	6.4	1.7	5.3	1.6	1	0.26	166	0.47
14	14	15	5.3	1.6	4.1	1.5	1	1.6	166	3
15	15	1	4.1	1.5	3.2	2.4	1	2	0	3.6
16	1	16	3.2	2.4	6.8	1.4	1	2.3	79	4.1
17	16	17	6.8	1.4	6.9	0.94	1	2.6	73	4.8
18	17	18	6.9	0.94	7.2	1.5	3	1.6	44	2
19	18	19	7.2	1.5	8	8.1	1	1.3	0	2.4
20	19	20	8	8.1	10	7	2	0.67	0	0.93
21	20	21	10	7	6	4	1	2.2	277	3.9
22	21	1	6	4	3.2	2.4	1	2.6	0	4.7
23	1	22	3.2	2.4	7.8	2.6	1	1.4	399	2.6
24	22	23	7.8	2.6	7.7	2.8	1	5.5	84	10
25	23	24	7.7	2.8	7	2.8	1	5.8	133	11
26	24	25	7	2.8	7	2.7	3	3.9	166	4.7
27	25	26	7	2.7	7	4.6	1	0.43	177	0.77
28	26	27	7	4.6	8.6	6.2	1	3.3	211	5.9
29	27	28	8.6	6.2	9.6	8.4	1	0.61	477	1.1
30	28	29	9.6	8.4	9.3	8.6	2	0.49	200	0.69
31	29	30	9.3	8.6	8.4	7.9	1	1.9	188	3.5
32	30	31	8.4	7.9	9.3	7.5	1	3.4	144	6.2
33	31	32	9.3	7.5	7.1	6.2	1	4.6	199	8.4
34	32	1	7.1	6.2	3.2	2.4	1	4.1	0	7.4
35	1	33	3.2	2.4	5.5	2	1	3.5	9.9	6.3
36	33	34	5.5	2	5.9	1.8	1	1.8	67	3.2
37	34	35	5.9	1.8	6.1	2.8	1	3.1	0	5.7
38	35	36	6.1	2.8	14	6.1	1	3.7	411	6.8
39	36	37	14	6.1	12	4.8	1	2.6	244	4.8

Table A.1: Pre-existing topology Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Cables	Cost (M Dkk)	Pks (kW)	Dist (km)
40	37	38	12	4.8	11	5	1	2.5	288	4.5
41	38	39	11	5	9.3	5.9	1	4	422	7.3
42	39	40	9.3	5.9	6	3.3	3	5.4	64	6.6
43	40	41	6	3.3	7	3.9	3	3	433	3.6
44	41	42	7	3.9	7.5	3.6	3	5.4	6.4	6.6
45	42	43	7.5	3.6	8.2	3.8	3	5.3	0	6.4
46	43	44	8.2	3.8	8.5	4	1	1.2	244	2.2
47	44	45	8.5	4	9.1	3.9	1	6.8	388	12
48	45	46	9.1	3.9	10	4.1	4	7.2	311	8.2
49	46	1	10	4.1	3.2	2.4	1	7.9	0	14
50	1	47	3.2	2.4	5.7	2.4	1	4.8	399	8.7
51	47	48	5.7	2.4	5.9	3.6	1	6.3	97	11
52	48	49	5.9	3.6	6.3	4.1	1	2.2	133	3.9
53	49	50	6.3	4.1	6.1	4.3	2	3.9	199	5.4
54	50	51	6.1	4.3	6.1	4.7	1	2	15	3.7
55	51	52	6.1	4.7	6.4	6.1	1	1.3	111	2.4
56	52	53	6.4	6.1	7.8	6.2	1	2.9	188	5.3
57	53	54	7.8	6.2	7.3	5.8	1	3.8	200	7
58	54	55	7.3	5.8	8.1	5.5	1	2	111	3.7
59	55	56	8.1	5.5	6.7	1.9	1	1.1	2.6	1.9
60	56	57	6.7	1.9	7.5	1.4	1	0.31	84	0.56
61	57	58	7.5	1.4	7.6	1.4	3	0.58	17	0.71
62	58	59	7.6	1.4	7.4	1.8	3	3.4	27	4.1
63	59	60	7.4	1.8	7.1	1.8	3	1.7	75	2
64	60	1	7.1	1.8	3.2	2.4	1	2.4	0	4.4

Table A.2: Pre-existing topology

Links	Sub1	Sub2	X1	Y1	X2	Y2	Cables	Cost (M Dkk)	Pks (kW)	Dist (km)	rRI	tRI
1	1	23	3.2	2.4	3.5	0.57	1	2	111	3.6	0.95	0.28
2	23	24	3.5	0.57	4.1	0.87	1	0.76	100	1.4	0.98	0.29
3	24	22	4.1	0.87	4.1	1.5	1	0.7	166	1.3	0.98	0.3
4	22	19	4.1	1.5	5.3	1.6	1	1.3	166	2.4	0.97	0.31
5	19	9	5.3	1.6	6	1.1	1	0.92	97	1.7	0.97	0.32
6	9	10	6	1.1	5.9	0.66	1	0.52	85	0.94	0.98	0.33
7	10	3	5.9	0.66	6.2	0.78	1	0.38	18	0.68	0.98	0.33
8	3	12	6.2	0.78	7.1	1.1	1	1.1	34	1.9	0.97	0.34
9	12	4	7.1	1.1	7.2	1.5	1	0.44	44	0.8	0.98	0.35
10	4	6	7.2	1.5	7.6	1.4	1	0.41	17	0.74	0.98	0.36
11	6	8	7.6	1.4	7.4	1.8	1	0.43	27	0.79	0.98	0.36
12	8	2	7.4	1.8	7.1	1.8	1	0.33	75	0.61	0.98	0.37
13	2	33	7.1	1.8	6.7	1.9	1	0.45	2.6	0.83	0.98	0.38
14	33	15	6.7	1.9	6.4	1.7	1	0.3	144	0.54	0.98	0.38
15	15	42	6.4	1.7	5.9	1.8	1	0.61	9.9	1.1	0.98	0.39
16	42	17	5.9	1.8	5.8	2	1	0.25	199	0.46	0.99	0.4
17	17	31	5.8	2	5.5	2	1	0.29	67	0.52	0.98	0.4
18	31	39	5.5	2	5.7	2.4	1	0.48	399	0.87	0.98	0.41
19	39	29	5.7	2.4	7	2.7	1	1.5	133	2.7	0.96	0.42
20	29	43	7	2.7	7.8	2.4	1	0.95	277	1.7	0.97	0.43
21	43	25	7.8	2.4	7	2.8	1	0.95	84	1.7	0.97	0.44
22	25	36	7	2.8	6	3.3	1	1.3	64	2.3	0.97	0.46
23	36	11	6	3.3	5.9	3.6	1	0.31	97	0.57	0.98	0.47
24	11	41	5.9	3.6	6.1	4.3	1	0.77	199	1.4	0.98	0.48
25	41	16	6.1	4.3	6.1	4.7	1	0.48	15	0.88	0.98	0.49
26	16	14	6.1	4.7	6.3	4.1	1	0.76	133	1.4	0.98	0.5
27	14	44	6.3	4.1	7	3.9	1	0.74	433	1.3	0.98	0.51
28	44	45	7	3.9	7.5	3.6	1	0.62	6.4	1.1	0.98	0.53
29	45	28	7.5	3.6	9.1	3.9	1	1.9	388	3.4	0.96	0.54
30	28	27	9.1	3.9	10	4.1	1	1.2	311	2.2	0.97	0.56
31	27	5	10	4.1	7	4.6	1	3.5	166	6.4	0.93	0.58
32	5	13	7	4.6	7.3	5.8	1	1.3	200	2.4	0.97	0.62
33	13	20	7.3	5.8	6.4	6.1	1	1	111	1.8	0.97	0.65
34	20	21	6.4	6.1	7.1	6.2	1	0.85	144	1.5	0.97	0.66
35	21	32	7.1	6.2	7.8	6.2	1	0.75	188	1.4	0.98	0.68
36	32	35	7.8	6.2	8.1	5.5	1	0.85	111	1.5	0.97	0.7
37	35	30	8.1	5.5	9.3	7.5	1	2.7	188	4.9	0.94	0.72
38	30	34	9.3	7.5	9.6	8.4	1	0.96	211	1.7	0.97	0.76
39	34	37	9.6	8.4	9.3	8.6	1	0.38	477	0.69	0.98	0.78
40	37	38	9.3	8.6	8.4	7.9	1	1.2	200	2.2	0.97	0.79
41	38	40	8.4	7.9	9.3	5.9	1	2.5	422	4.5	0.95	0.82
42	40	26	9.3	5.9	11	5	1	2	288	3.6	0.95	0.87
43	26	7	11	5	12	4.8	1	1.5	244	2.7	0.96	0.91
44	7	18	12	4.8	14	6.1	1	2.1	411	3.8	0.95	0.94

Table A.3: Case 1 : 2 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Cables	Cost (M Dkk)	Pks (kW)	Dist (km)	rRI	tRI
1	1	23	3.2	2.4	3.5	0.57	1	2	111	3.6	0.95	0.28
2	23	24	3.5	0.57	4.1	0.87	1	0.76	100	1.4	0.98	0.29
3	24	22	4.1	0.87	4.1	1.5	1	0.7	166	1.3	0.98	0.3
4	22	19	4.1	1.5	5.3	1.6	2	1.7	166	2.4	0.97	0.3
5	19	9	5.3	1.6	6	1.1	1	0.92	97	1.7	0.97	0.32
6	9	10	6	1.1	5.9	0.66	1	0.52	85	0.94	0.98	0.32
7	10	3	5.9	0.66	6.2	0.78	1	0.38	18	0.68	0.98	0.33
8	3	12	6.2	0.78	7.1	1.1	1	1.1	34	1.9	0.97	0.34
9	12	4	7.1	1.1	7.2	1.5	1	0.44	44	0.8	0.98	0.35
10	4	6	7.2	1.5	7.6	1.4	1	0.41	17	0.74	0.98	0.35
11	6	8	7.6	1.4	7.4	1.8	1	0.43	27	0.79	0.98	0.36
12	8	2	7.4	1.8	7.1	1.8	1	0.33	75	0.61	0.98	0.37
13	2	33	7.1	1.8	6.7	1.9	1	0.45	2.6	0.83	0.98	0.37
14	33	15	6.7	1.9	6.4	1.7	1	0.3	144	0.54	0.98	0.38
15	15	42	6.4	1.7	5.9	1.8	1	0.61	9.9	1.1	0.98	0.38
16	42	17	5.9	1.8	5.8	2	1	0.25	199	0.46	0.99	0.39
17	17	31	5.8	2	5.5	2	1	0.29	67	0.52	0.98	0.4
18	31	39	5.5	2	5.7	2.4	1	0.48	399	0.87	0.98	0.4
19	39	29	5.7	2.4	7	2.7	3	2.1	133	2.7	0.96	0.41
20	29	43	7	2.7	7.8	2.4	1	0.95	277	1.7	0.97	0.43
21	43	25	7.8	2.4	7	2.8	2	1.2	84	1.7	0.97	0.44
22	25	36	7	2.8	6	3.3	1	1.3	64	2.3	0.97	0.45
23	36	11	6	3.3	5.9	3.6	1	0.31	97	0.57	0.98	0.47
24	11	41	5.9	3.6	6.1	4.3	1	0.69	133	1.3	0.98	0.47
25	41	16	6.1	4.3	6.1	4.7	1	0.35	199	0.64	0.98	0.48
26	16	14	6.1	4.7	6.3	4.1	1	0.48	15	0.88	0.98	0.49
27	14	44	6.3	4.1	7	3.9	1	4.6	311	8.3	0.91	0.5
28	44	45	7	3.9	7.5	3.6	2	1.6	388	2.2	0.97	0.55
29	45	28	7.5	3.6	9.1	3.9	4	3	6.4	3.4	0.96	0.57
30	28	27	9.1	3.9	10	4.1	1	0.62	433	1.1	0.98	0.59
31	27	5	10	4.1	7	4.6	2	1	166	1.4	0.98	0.61
32	5	13	7	4.6	7.3	5.8	1	1.3	200	2.4	0.97	0.62
33	13	20	7.3	5.8	6.4	6.1	2	1.3	111	1.8	0.97	0.64
34	20	21	6.4	6.1	7.1	6.2	1	0.85	144	1.5	0.97	0.66
35	21	32	7.1	6.2	7.8	6.2	1	1.3	111	2.5	0.97	0.68
36	32	35	7.8	6.2	8.1	5.5	1	0.85	188	1.5	0.97	0.7
37	35	30	8.1	5.5	9.3	7.5	2	2.9	188	4.1	0.95	0.72
38	30	34	9.3	7.5	9.6	8.4	1	0.96	211	1.7	0.97	0.76
39	34	37	9.6	8.4	9.3	8.6	1	0.38	477	0.69	0.98	0.78
40	37	38	9.3	8.6	8.4	7.9	3	1.7	200	2.2	0.97	0.79
41	38	40	8.4	7.9	9.3	5.9	3	3.5	422	4.5	0.95	0.82
42	40	26	9.3	5.9	11	5	4	3.2	288	3.6	0.95	0.87
43	26	7	11	5	12	4.8	3	2.1	244	2.7	0.96	0.91
44	7	18	12	4.8	14	6.1	3	2.9	411	3.8	0.95	0.94

Table A.4: Case 2 : 2 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Cables	Cost (M Dkk)	Pks (kW)	Dist (km)	rRI	tRI
1	1	22	3.2	2.4	4.1	1.5	166	2.4	1	1.3	0.97	0.33
2	22	24	4.1	1.5	4.1	0.87	100	1.3	1	0.7	0.98	0.34
3	24	23	4.1	0.87	3.5	0.57	111	1.4	1	0.76	0.98	0.35
4	22	19	4.1	1.5	5.3	1.6	166	2.4	1	1.3	0.97	0.36
5	19	31	5.3	1.6	5.5	2	67	0.91	1	0.5	0.98	0.37
6	31	17	5.5	2	5.8	2	199	0.52	1	0.29	0.98	0.38
7	17	42	5.8	2	5.9	1.8	9.9	0.46	1	0.25	0.99	0.39
8	17	39	5.8	2	5.7	2.4	399	0.8	1	0.44	0.98	0.39
9	42	15	5.9	1.8	6.4	1.7	144	1.1	1	0.61	0.98	0.4
10	15	33	6.4	1.7	6.7	1.9	2.6	0.54	1	0.3	0.98	0.41
11	33	2	6.7	1.9	7.1	1.8	75	0.83	1	0.45	0.98	0.41
12	2	8	7.1	1.8	7.4	1.8	27	0.61	1	0.33	0.98	0.42
13	8	4	7.4	1.8	7.2	1.5	44	0.66	1	0.36	0.98	0.43
14	4	6	7.2	1.5	7.6	1.4	17	0.74	1	0.41	0.98	0.44
15	4	12	7.2	1.5	7.1	1.1	34	0.8	1	0.44	0.98	0.44
16	42	9	5.9	1.8	6	1.1	97	1.4	1	0.78	0.98	0.45
17	9	3	6	1.1	6.2	0.78	18	0.81	1	0.45	0.98	0.46
18	3	10	6.2	0.78	5.9	0.66	85	0.68	1	0.38	0.98	0.47
19	33	29	6.7	1.9	7	2.7	133	1.8	1	0.98	0.97	0.48
20	29	25	7	2.7	7	2.8	84	0.31	1	0.17	0.99	0.49
21	29	43	7	2.7	7.8	2.4	277	1.7	1	0.95	0.97	0.5
22	39	36	5.7	2.4	6	3.3	64	2.1	1	1.2	0.97	0.51
23	36	11	6	3.3	5.9	3.6	97	0.57	1	0.31	0.98	0.53
24	11	14	5.9	3.6	6.3	4.1	133	1.3	1	0.69	0.98	0.54
25	14	41	6.3	4.1	6.1	4.3	199	0.64	1	0.35	0.98	0.55
26	41	16	6.1	4.3	6.1	4.7	15	0.88	1	0.48	0.98	0.56
27	14	44	6.3	4.1	7	3.9	433	1.3	1	0.74	0.98	0.57
28	44	45	7	3.9	7.5	3.6	6.4	1.1	1	0.62	0.98	0.58
29	44	5	7	3.9	7	4.6	166	1.4	1	0.78	0.98	0.6
30	5	13	7	4.6	7.3	5.8	200	2.4	1	1.3	0.97	0.61
31	13	21	7.3	5.8	7.1	6.2	144	0.96	1	0.53	0.98	0.63
32	13	32	7.3	5.8	7.8	6.2	188	1.3	1	0.74	0.98	0.65
33	32	35	7.8	6.2	8.1	5.5	111	1.5	1	0.85	0.97	0.66
34	21	20	7.1	6.2	6.4	6.1	111	1.5	1	0.85	0.97	0.68
35	35	40	8.1	5.5	9.3	5.9	422	2.7	1	1.5	0.96	0.7
36	40	30	9.3	5.9	9.3	7.5	188	3.4	1	1.9	0.96	0.72
37	30	34	9.3	7.5	9.6	8.4	211	1.7	1	0.96	0.97	0.75
38	34	37	9.6	8.4	9.3	8.6	477	0.69	1	0.38	0.98	0.77
39	30	38	9.3	7.5	8.4	7.9	200	2.1	1	1.1	0.97	0.79
40	45	28	7.5	3.6	9.1	3.9	388	3.4	1	1.9	0.96	0.81
41	28	27	9.1	3.9	10	4.1	311	2.2	1	1.2	0.97	0.85
42	40	26	9.3	5.9	11	5	288	3.6	1	2	0.95	0.88
43	26	7	11	5	12	4.8	244	2.7	1	1.5	0.96	0.92
44	7	18	12	4.8	14	6.1	411	3.8	1	2.1	0.95	0.95

Table A.5: Case 1 : 3 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Cables	Cost (M Dkk)	Pks (kW)	Dist (km)	rRI	tRI
1	1	22	3.2	2.4	4.1	1.5	166	2.4	1	1.3	0.97	0.33
2	22	24	4.1	1.5	4.1	0.87	100	1.3	1	0.7	0.98	0.34
3	24	23	4.1	0.87	3.5	0.57	111	1.4	1	0.76	0.98	0.35
4	22	19	4.1	1.5	5.3	1.6	166	2.4	1	1.3	0.97	0.36
5	19	31	5.3	1.6	5.5	2	67	0.91	1	0.5	0.98	0.37
6	31	17	5.5	2	5.8	2	199	0.52	1	0.29	0.98	0.38
7	17	42	5.8	2	5.9	1.8	9.9	0.46	1	0.25	0.99	0.39
8	17	39	5.8	2	5.7	2.4	399	0.8	4	0.44	0.98	0.39
9	42	15	5.9	1.8	6.4	1.7	144	1.1	1	0.61	0.98	0.4
10	15	33	6.4	1.7	6.7	1.9	2.6	0.54	1	0.3	0.98	0.41
11	33	2	6.7	1.9	7.1	1.8	75	0.83	1	0.45	0.98	0.41
12	2	8	7.1	1.8	7.4	1.8	27	0.61	1	0.33	0.98	0.42
13	8	4	7.4	1.8	7.2	1.5	44	0.66	1	0.36	0.98	0.43
14	4	6	7.2	1.5	7.6	1.4	17	0.74	1	0.41	0.98	0.44
15	4	12	7.2	1.5	7.1	1.1	34	0.8	1	0.44	0.98	0.44
16	42	9	5.9	1.8	6	1.1	97	1.4	1	0.78	0.98	0.45
17	9	3	6	1.1	6.2	0.78	18	0.81	1	0.45	0.98	0.46
18	3	10	6.2	0.78	5.9	0.66	85	0.68	1	0.38	0.98	0.47
19	33	29	6.7	1.9	7	2.7	133	1.8	1	0.98	0.97	0.48
20	29	25	7	2.7	7	2.8	84	0.31	1	0.17	0.99	0.49
21	29	43	7	2.7	7.8	2.4	277	1.7	2	0.95	0.97	0.5
22	25	36	7	2.8	6	3.3	64	2.3	1	1.3	0.97	0.51
23	36	11	6	3.3	5.9	3.6	97	0.57	1	0.31	0.98	0.53
24	11	14	5.9	3.6	6.3	4.1	133	1.3	1	0.69	0.98	0.54
25	14	41	6.3	4.1	6.1	4.3	199	0.64	1	0.35	0.98	0.55
26	41	16	6.1	4.3	6.1	4.7	15	0.88	1	0.48	0.98	0.56
27	14	44	6.3	4.1	7	3.9	433	1.3	4	0.74	0.98	0.57
28	44	45	7	3.9	7.5	3.6	6.4	1.1	1	0.62	0.98	0.58
29	44	5	7	3.9	7	4.6	166	1.4	1	0.78	0.98	0.6
30	5	13	7	4.6	7.3	5.8	200	2.4	1	1.3	0.97	0.61
31	13	21	7.3	5.8	7.1	6.2	144	0.96	1	0.53	0.98	0.63
32	13	32	7.3	5.8	7.8	6.2	188	1.3	1	0.74	0.98	0.65
33	32	35	7.8	6.2	8.1	5.5	111	1.5	1	0.85	0.97	0.66
34	21	20	7.1	6.2	6.4	6.1	111	1.5	1	0.85	0.97	0.68
35	35	40	8.1	5.5	9.3	5.9	422	2.7	4	1.5	0.96	0.7
36	45	28	7.5	3.6	9.1	3.9	388	3.4	4	1.9	0.96	0.72
37	28	27	9.1	3.9	10	4.1	311	2.2	3	1.2	0.97	0.75
38	40	30	9.3	5.9	9.3	7.5	188	3.4	1	1.9	0.96	0.78
39	30	34	9.3	7.5	9.6	8.4	211	1.7	1	0.96	0.97	0.81
40	34	37	9.6	8.4	9.3	8.6	477	0.69	4	0.38	0.98	0.84
41	30	38	9.3	7.5	8.4	7.9	200	2.1	1	1.1	0.97	0.85
42	40	26	9.3	5.9	11	5	288	3.6	2	2	0.95	0.88
43	26	7	11	5	12	4.8	244	2.7	2	1.5	0.96	0.92
44	7	18	12	4.8	14	6.1	411	3.8	4	2.1	0.95	0.95

Table A.6: Case 2 : 3 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	24	3.2	2.4	4.1	0.87	288	3.5	1	4.4
2	24	23	4.1	0.87	3.5	0.57	122	1.4	1	1.5
3	1	22	3.2	2.4	4.1	1.5	488	2.4	1	3.1
4	22	19	4.1	1.5	5.3	1.6	288	2.4	2	3.5
5	19	9	5.3	1.6	6	1.1	111	1.7	1	1.8
6	1	10	3.2	2.4	5.9	0.66	377	6.3	1	8
7	10	3	5.9	0.66	6.2	0.78	288	0.68	1	0.75
8	3	12	6.2	0.78	7.1	1.1	266	1.9	1	2.1
9	12	4	7.1	1.1	7.2	1.5	211	0.8	1	0.88
10	4	6	7.2	1.5	7.6	1.4	166	0.74	1	0.82
11	6	8	7.6	1.4	7.4	1.8	133	0.79	1	0.87
12	8	2	7.4	1.8	7.1	1.8	94	0.61	1	0.67
13	1	17	3.2	2.4	5.8	2	399	5.2	1	6.8
14	17	42	5.8	2	5.9	1.8	200	0.46	1	0.5
15	42	15	5.9	1.8	6.4	1.7	200	1.1	1	1.2
16	15	33	6.4	1.7	6.7	1.9	2.6	0.54	1	0.59
17	1	31	3.2	2.4	5.5	2	877	4.6	1	5.1
18	31	39	5.5	2	5.7	2.4	800	0.87	1	1.1
19	39	29	5.7	2.4	7	2.7	400	2.7	3	4
20	29	43	7	2.7	7.8	2.4	277	1.7	1	2.2
21	1	25	3.2	2.4	7	2.8	633	7.7	1	10
22	25	36	7	2.8	6	3.3	544	2.3	1	2.5
23	36	11	6	3.3	5.9	3.6	488	0.57	1	0.63
24	11	14	5.9	3.6	6.3	4.1	366	1.3	1	1.4
25	14	41	6.3	4.1	6.1	4.3	222	0.64	1	0.7
26	1	16	3.2	2.4	6.1	4.7	2.310 ³	9.4	1	10
27	16	44	6.1	4.7	7	3.9	2.310 ³	2.4	1	3.2
28	44	45	7	3.9	7.5	3.6	1.910 ³	1.1	2	1.4
29	45	28	7.5	3.6	9.1	3.9	1.910 ³	3.4	1	4.8
30	28	27	9.1	3.9	10	4.1	1.510 ³	2.2	3	3.3
31	27	5	10	4.1	7	4.6	1.110 ³	6.4	4	11
32	5	13	7	4.6	7.3	5.8	999	2.4	1	3
33	13	20	7.3	5.8	6.4	6.1	788	1.8	2	2.3
34	20	21	6.4	6.1	7.1	6.2	644	1.5	1	1.7
35	21	32	7.1	6.2	7.8	6.2	499	1.4	1	1.5
36	32	35	7.8	6.2	8.1	5.5	311	1.5	1	1.7
37	35	30	8.1	5.5	9.3	7.5	200	4.9	2	7.3
38	1	34	3.2	2.4	9.6	8.4	2.310 ³	19	1	27
39	34	37	9.6	8.4	9.3	8.6	2.110 ³	0.69	1	0.87
40	37	38	9.3	8.6	8.4	7.9	1.610 ³	2.2	4	3.6
41	38	40	8.4	7.9	9.3	5.9	1.310 ³	4.5	4	8
42	40	26	9.3	5.9	11	5	922	3.6	4	6
43	26	7	11	5	12	4.8	666	2.7	3	4
44	7	18	12	4.8	14	6.1	411	3.8	3	6.2
45	2	41	7.1	1.8	6.1	4.3	199	0.83	4	0.73
46	33	43	6.7	1.9	7.8	2.4	277	5.1	4	4.4
47	9	30	6	1.1	9.3	7.5	188	5.1	4	4.5
48	23	18	3.5	0.57	14	6.1	411	9	4	7.9

Table A.7: Case 4 : Peaks occur at the same time - 2 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	23	3.2	2.4	3.5	0.57	344	3.6	1	4
2	23	24	3.5	0.57	4.1	0.87	288	1.4	1	1.5
3	24	22	4.1	0.87	4.1	1.5	200	1.3	1	1.4
4	22	19	4.1	1.5	5.3	1.6	100	2.4	1	2.7
5	1	9	3.2	2.4	6	1.1	233	6.1	1	6.7
6	9	10	6	1.1	5.9	0.66	200	0.94	1	1
7	10	3	5.9	0.66	6.2	0.78	133	0.68	1	0.75
8	3	12	6.2	0.78	7.1	1.1	122	1.9	1	2.1
9	12	4	7.1	1.1	7.2	1.5	100	0.8	1	0.88
10	4	6	7.2	1.5	7.6	1.4	78	0.74	1	0.82
11	6	8	7.6	1.4	7.4	1.8	70	0.79	1	0.87
12	8	2	7.4	1.8	7.1	1.8	52	0.61	1	0.67
13	2	33	7.1	1.8	6.7	1.9	0.48	0.83	1	0.91
14	1	39	3.2	2.4	5.7	2.4	211	4.9	1	5.4
15	39	31	5.7	2.4	5.5	2	166	0.87	1	0.96
16	31	17	5.5	2	5.8	2	144	0.52	1	0.57
17	17	42	5.8	2	5.9	1.8	100	0.46	1	0.5
18	42	15	5.9	1.8	6.4	1.7	98	1.1	1	1.2
19	1	36	3.2	2.4	6	3.3	111	6.6	1	7.3
20	36	25	6	3.3	7	2.8	95	2.3	1	2.5
21	25	29	7	2.8	7	2.7	72	0.31	1	0.34
22	29	43	7	2.7	7.8	2.4	36	1.7	1	1.9
23	1	11	3.2	2.4	5.9	3.6	688	7.2	1	7.9
24	11	14	5.9	3.6	6.3	4.1	666	1.3	1	1.4
25	14	44	6.3	4.1	7	3.9	622	1.3	1	1.7
26	44	45	7	3.9	7.5	3.6	333	1.1	1	1.2
27	45	28	7.5	3.6	9.1	3.9	333	3.4	1	4.3
28	28	27	9.1	3.9	10	4.1	222	2.2	1	2.4
29	27	5	10	4.1	7	4.6	93	6.4	3	8.5
30	5	16	7	4.6	6.1	4.7	65	1.9	1	2.1
31	16	41	6.1	4.7	6.1	4.3	63	0.88	1	0.97
32	1	20	3.2	2.4	6.4	6.1	344	12	1	13
33	20	21	6.4	6.1	7.1	6.2	322	1.5	1	1.7
34	21	13	7.1	6.2	7.3	5.8	299	0.96	1	1.1
35	13	35	7.3	5.8	8.1	5.5	266	1.7	1	1.9
36	35	32	8.1	5.5	7.8	6.2	244	1.5	1	1.7
37	32	30	7.8	6.2	9.3	7.5	222	4.1	1	4.5
38	30	34	9.3	7.5	9.6	8.4	188	1.7	1	1.9
39	34	37	9.6	8.4	9.3	8.6	133	0.69	1	0.76
40	37	38	9.3	8.6	8.4	7.9	56	2.2	1	2.5
41	1	40	3.2	2.4	9.3	5.9	277	15	1	22
42	40	26	9.3	5.9	11	5	199	3.6	1	4
43	26	7	11	5	12	4.8	133	2.7	1	3
44	7	18	12	4.8	14	6.1	55	3.8	1	4.2
45	33	41	6.7	1.9	6.1	4.3	199	0.54	4	0.47
46	15	38	6.4	1.7	8.4	7.9	200	2.8	4	2.5
47	19	18	5.3	1.6	14	6.1	411	5.1	4	4.4
48	43	33	7.8	2.4	6.7	1.9	2.6	11	4	9.7

Table A.8: Case 4 : Christmas- 2 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	24	3.2	2.4	4.1	0.87	133	3.5	1	3.8
2	24	23	4.1	0.87	3.5	0.57	58	1.4	1	1.5
3	1	22	3.2	2.4	4.1	1.5	200	2.4	1	2.7
4	22	19	4.1	1.5	5.3	1.6	80	2.4	1	2.7
5	1	9	3.2	2.4	6	1.1	199	6.1	1	7.7
6	9	10	6	1.1	5.9	0.66	122	0.94	1	1
7	10	3	5.9	0.66	6.2	0.78	74	0.68	1	0.75
8	3	12	6.2	0.78	7.1	1.1	65	1.9	1	2.1
9	12	4	7.1	1.1	7.2	1.5	44	0.8	1	0.88
10	4	6	7.2	1.5	7.6	1.4	5.2	0.74	1	0.82
11	1	17	3.2	2.4	5.8	2	311	5.2	1	6.5
12	17	42	5.8	2	5.9	1.8	199	0.46	1	0.5
13	42	15	5.9	1.8	6.4	1.7	188	1.1	1	1.2
14	15	33	6.4	1.7	6.7	1.9	66	0.54	1	0.59
15	33	2	6.7	1.9	7.1	1.8	65	0.83	1	0.91
16	2	8	7.1	1.8	7.4	1.8	19	0.61	1	0.67
17	1	31	3.2	2.4	5.5	2	522	4.6	1	5.1
18	31	39	5.5	2	5.7	2.4	488	0.87	1	0.96
19	39	29	5.7	2.4	7	2.7	266	2.7	2	3.4
20	29	43	7	2.7	7.8	2.4	177	1.7	1	1.9
21	1	11	3.2	2.4	5.9	3.6	200	7.2	1	9.1
22	11	36	5.9	3.6	6	3.3	122	0.57	1	0.63
23	36	25	6	3.3	7	2.8	79	2.3	1	2.5
24	1	41	3.2	2.4	6.1	4.3	1.210 ³	8.5	1	12
25	41	16	6.1	4.3	6.1	4.7	1.110 ³	0.88	1	0.97
26	16	14	6.1	4.7	6.3	4.1	110 ³	1.4	1	1.5
27	14	44	6.3	4.1	7	3.9	988	1.3	1	1.5
28	44	45	7	3.9	7.5	3.6	777	1.1	1	1.2
29	45	28	7.5	3.6	9.1	3.9	777	3.4	1	4.5
30	28	27	9.1	3.9	10	4.1	566	2.2	2	3.1
31	27	5	10	4.1	7	4.6	400	6.4	3	9.5
32	5	13	7	4.6	7.3	5.8	311	2.4	1	2.6
33	13	21	7.3	5.8	7.1	6.2	199	0.96	1	1.1
34	21	20	7.1	6.2	6.4	6.1	85	1.5	1	1.7
35	1	32	3.2	2.4	7.8	6.2	888	13	1	19
36	32	35	7.8	6.2	8.1	5.5	799	1.5	1	1.7
37	35	30	8.1	5.5	9.3	7.5	711	4.9	1	6.2
38	30	34	9.3	7.5	9.6	8.4	588	1.7	1	1.9
39	34	37	9.6	8.4	9.3	8.6	411	0.69	1	0.87
40	1	38	3.2	2.4	8.4	7.9	1.110 ³	24	1	34
41	38	40	8.4	7.9	9.3	5.9	966	4.5	3	7.5
42	40	26	9.3	5.9	11	5	666	3.6	4	5.8
43	26	7	11	5	12	4.8	499	2.7	2	3.9
44	7	18	12	4.8	14	6.1	244	3.8	3	5.8
45	25	23	7	2.8	3.5	0.57	111	0.79	4	0.69
46	43	20	7.8	2.4	6.4	6.1	111	1.7	4	1.5
47	8	37	7.4	1.8	9.3	8.6	477	4.1	4	3.6
48	6	18	7.6	1.4	14	6.1	411	6.6	4	5.8
49	19	25	5.3	1.6	7	2.8	84	9.9	4	8.8

Table A.9: Case 4 : Summer - 2 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	23	3.2	2.4	3.5	0.57	422	3.6	1	4
2	23	24	3.5	0.57	4.1	0.87	333	1.4	1	1.5
3	24	22	4.1	0.87	4.1	1.5	244	1.3	1	1.4
4	22	19	4.1	1.5	5.3	1.6	133	2.4	1	2.7
5	1	9	3.2	2.4	6	1.1	122	6.1	1	6.7
6	9	10	6	1.1	5.9	0.66	72	0.94	1	1
7	1	17	3.2	2.4	5.8	2	333	5.2	1	5.7
8	17	42	5.8	2	5.9	1.8	300	0.46	1	0.5
9	42	15	5.9	1.8	6.4	1.7	299	1.1	1	1.2
10	15	33	6.4	1.7	6.7	1.9	177	0.54	1	0.59
11	33	2	6.7	1.9	7.1	1.8	177	0.83	1	0.91
12	2	8	7.1	1.8	7.4	1.8	99	0.61	1	0.67
13	8	6	7.4	1.8	7.6	1.4	77	0.79	1	0.87
14	6	4	7.6	1.4	7.2	1.5	67	0.74	1	0.82
15	4	12	7.2	1.5	7.1	1.1	34	0.8	1	0.88
16	12	3	7.1	1.1	6.2	0.78	13	1.9	1	2.1
17	1	31	3.2	2.4	5.5	2	97	4.6	1	5.1
18	31	39	5.5	2	5.7	2.4	70	0.87	1	0.96
19	1	29	3.2	2.4	7	2.7	91	7.5	1	8.3
20	29	43	7	2.7	7.8	2.4	45	1.7	1	1.9
21	1	25	3.2	2.4	7	2.8	777	7.7	1	8.5
22	25	36	7	2.8	6	3.3	722	2.3	1	2.5
23	36	11	6	3.3	5.9	3.6	700	0.57	1	0.63
24	11	14	5.9	3.6	6.3	4.1	688	1.3	1	1.4
25	14	44	6.3	4.1	7	3.9	644	1.3	1	1.7
26	44	45	7	3.9	7.5	3.6	299	1.1	2	1.4
27	45	5	7.5	3.6	7	4.6	288	2.2	1	2.5
28	5	28	7	4.6	9.1	3.9	244	4.4	1	5.6
29	28	27	9.1	3.9	10	4.1	133	2.2	1	2.4
30	1	41	3.2	2.4	6.1	4.3	777	8.5	1	11
31	41	16	6.1	4.3	6.1	4.7	699	0.88	1	0.97
32	16	20	6.1	4.7	6.4	6.1	688	2.8	1	3.1
33	20	21	6.4	6.1	7.1	6.2	633	1.5	1	1.7
34	21	13	7.1	6.2	7.3	5.8	599	0.96	1	1.1
35	13	35	7.3	5.8	8.1	5.5	566	1.7	1	1.9
36	35	32	8.1	5.5	7.8	6.2	533	1.5	1	1.7
37	32	30	7.8	6.2	9.3	7.5	500	4.1	1	4.5
38	30	34	9.3	7.5	9.6	8.4	466	1.7	1	1.9
39	34	37	9.6	8.4	9.3	8.6	400	0.69	1	0.87
40	1	38	3.2	2.4	8.4	7.9	422	24	1	34
41	38	40	8.4	7.9	9.3	5.9	344	4.5	2	6.5
42	40	26	9.3	5.9	11	5	277	3.6	1	4
43	26	7	11	5	12	4.8	199	2.7	1	3
44	7	18	12	4.8	14	6.1	70	3.8	2	4.8
45	3	43	6.2	0.78	7.8	2.4	277	2.2	4	2
46	10	27	5.9	0.66	10	4.1	311	3.4	4	3
47	19	37	5.3	1.6	9.3	8.6	477	9.9	4	8.8
48	39	18	5.7	2.4	14	6.1	411	10	4	8.8

Table A.10: Case 4 : New year's eve- 2 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	24	3.2	2.4	4.1	0.87	233	3.5	1	4.4
2	24	23	4.1	0.87	3.5	0.57	100	1.4	1	1.5
3	1	22	3.2	2.4	4.1	1.5	377	2.4	1	3.1
4	22	19	4.1	1.5	5.3	1.6	200	2.4	2	3.1
5	19	31	5.3	1.6	5.5	2	67	0.91	1	1
6	1	39	3.2	2.4	5.7	2.4	788	4.9	1	7
7	39	17	5.7	2.4	5.8	2	422	0.8	1	0.88
8	17	42	5.8	2	5.9	1.8	266	0.46	1	0.5
9	42	9	5.9	1.8	6	1.1	244	1.4	1	1.6
10	9	10	6	1.1	5.9	0.66	133	0.94	1	1
11	10	3	5.9	0.66	6.2	0.78	57	0.68	1	0.75
12	3	12	6.2	0.78	7.1	1.1	40	1.9	1	2.1
13	1	2	3.2	2.4	7.1	1.8	199	7.8	1	9.9
14	2	8	7.1	1.8	7.4	1.8	100	0.61	1	0.67
15	8	4	7.4	1.8	7.2	1.5	68	0.66	1	0.73
16	4	6	7.2	1.5	7.6	1.4	17	0.74	1	0.82
17	1	15	3.2	2.4	6.4	1.7	533	6.6	1	8.7
18	15	33	6.4	1.7	6.7	1.9	388	0.54	1	0.59
19	33	29	6.7	1.9	7	2.7	388	1.8	1	2
20	29	43	7	2.7	7.8	2.4	233	1.7	1	2.2
21	1	25	3.2	2.4	7	2.8	399	7.7	1	9.8
22	25	36	7	2.8	6	3.3	300	2.3	1	2.5
23	36	11	6	3.3	5.9	3.6	233	0.57	1	0.63
24	11	14	5.9	3.6	6.3	4.1	133	1.3	1	1.4
25	1	41	3.2	2.4	6.1	4.3	2.310 ³	8.5	1	12
26	41	16	6.1	4.3	6.1	4.7	2.110 ³	0.88	1	0.97
27	16	44	6.1	4.7	7	3.9	2.110 ³	2.4	1	3.2
28	44	45	7	3.9	7.5	3.6	1.710 ³	1.1	2	1.4
29	45	28	7.5	3.6	9.1	3.9	1.710 ³	3.4	1	4.8
30	28	27	9.1	3.9	10	4.1	1.410 ³	2.2	3	3.3
31	27	5	10	4.1	7	4.6	110 ³	6.4	4	11
32	5	13	7	4.6	7.3	5.8	888	2.4	1	3
33	13	20	7.3	5.8	6.4	6.1	699	1.8	1	2
34	20	21	6.4	6.1	7.1	6.2	577	1.5	1	1.7
35	21	32	7.1	6.2	7.8	6.2	422	1.4	1	1.5
36	32	35	7.8	6.2	8.1	5.5	288	1.5	1	1.7
37	35	30	8.1	5.5	9.3	7.5	188	4.9	2	7.3
38	1	34	3.2	2.4	9.6	8.4	2.110 ³	19	1	27
39	34	37	9.6	8.4	9.3	8.6	1.910 ³	0.69	1	0.87
40	37	38	9.3	8.6	8.4	7.9	1.410 ³	2.2	4	3.6
41	38	40	8.4	7.9	9.3	5.9	1.210 ³	4.5	3	7.5
42	40	26	9.3	5.9	11	5	811	3.6	4	6
43	26	7	11	5	12	4.8	533	2.7	3	4
44	7	18	12	4.8	14	6.1	377	3.8	2	6
45	12	43	7.1	1.1	7.8	2.4	277	1.2	4	1.1
46	6	23	7.6	1.4	3.5	0.57	111	4.4	4	3.9
47	31	30	5.5	2	9.3	7.5	188	4.9	4	4.4
48	14	18	6.3	4.1	14	6.1	411	9	4	7.9

Table A.11: Case 4 : Peaks occur at the same time- 2 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	22	3.2	2.4	4.1	1.5	200	2.4	1	1.3
2	22	24	4.1	1.5	4.1	0.87	166	1.3	1	0.7
3	1	23	3.2	2.4	3.5	0.57	122	3.6	1	2
4	23	19	3.5	0.57	5.3	1.6	177	4.1	1	2.3
5	19	31	5.3	1.6	5.5	2	67	0.91	1	0.5
6	31	17	5.5	2	5.8	2	199	0.52	1	0.29
7	1	39	3.2	2.4	5.7	2.4	400	4.9	4	4.3
8	39	42	5.7	2.4	5.9	1.8	9.9	1.2	1	0.68
9	42	15	5.9	1.8	6.4	1.7	200	1.1	1	0.61
10	15	33	6.4	1.7	6.7	1.9	2.6	0.54	1	0.3
11	1	9	3.2	2.4	6	1.1	111	6.1	1	3.3
12	9	12	6	1.1	7.1	1.1	43	2.3	1	1.3
13	12	4	7.1	1.1	7.2	1.5	62	0.8	1	0.44
14	4	8	7.2	1.5	7.4	1.8	36	0.66	1	0.36
15	8	2	7.4	1.8	7.1	1.8	94	0.61	1	0.33
16	4	6	7.2	1.5	7.6	1.4	17	0.74	1	0.41
17	1	3	3.2	2.4	6.2	0.78	20	6.8	1	3.7
18	1	10	3.2	2.4	5.9	0.66	89	6.3	1	3.5
19	10	29	5.9	0.66	7	2.7	144	4.6	1	2.5
20	29	25	7	2.7	7	2.8	97	0.31	1	0.17
21	29	43	7	2.7	7.8	2.4	277	1.7	2	1.2
22	25	36	7	2.8	6	3.3	64	2.3	1	1.3
23	36	11	6	3.3	5.9	3.6	122	0.57	1	0.31
24	11	14	5.9	3.6	6.3	4.1	133	1.3	1	0.69
25	14	41	6.3	4.1	6.1	4.3	222	0.64	1	0.35
26	1	44	3.2	2.4	7	3.9	433	8.6	4	7.6
27	44	45	7	3.9	7.5	3.6	6.4	1.1	1	0.62
28	44	5	7	3.9	7	4.6	166	1.4	1	0.78
29	5	16	7	4.6	6.1	4.7	15	1.9	1	1
30	5	13	7	4.6	7.3	5.8	211	2.4	1	1.3
31	13	21	7.3	5.8	7.1	6.2	177	0.96	1	0.53
32	13	32	7.3	5.8	7.8	6.2	188	1.3	1	0.74
33	32	35	7.8	6.2	8.1	5.5	111	1.5	1	0.85
34	21	20	7.1	6.2	6.4	6.1	133	1.5	1	0.85
35	35	40	8.1	5.5	9.3	5.9	422	2.7	4	2.3
36	45	28	7.5	3.6	9.1	3.9	388	3.4	4	3
37	28	27	9.1	3.9	10	4.1	377	2.2	3	1.8
38	1	30	3.2	2.4	9.3	7.5	200	17	1	9.5
39	30	34	9.3	7.5	9.6	8.4	244	1.7	2	1.2
40	34	37	9.6	8.4	9.3	8.6	511	0.69	4	0.61
41	30	38	9.3	7.5	8.4	7.9	244	2.1	2	1.5
42	34	26	9.6	8.4	11	5	288	7.4	2	5.3
43	26	7	11	5	12	4.8	244	2.7	2	1.9
44	7	18	12	4.8	14	6.1	411	3.8	4	3.3
45	24	3	4.1	0.87	6.2	0.78	18	2.1	4	3.7
46	17	41	5.8	2	6.1	4.3	199	4	4	14
47	33	27	6.7	1.9	10	4.1	311	7	4	43
48	6	18	7.6	1.4	14	6.1	411	20	4	344

Table A.12: Case 4 : Case 4 : Peaks occur at the same time- 3 Edges- 3 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	22	3.2	2.4	4.1	1.5	92	2.4	1	1.3
2	22	24	4.1	1.5	4.1	0.87	75	1.3	1	0.7
3	1	23	3.2	2.4	3.5	0.57	75	3.6	1	2
4	23	19	3.5	0.57	5.3	1.6	100	4.1	1	2.3
5	19	31	5.3	1.6	5.5	2	16	0.91	1	0.5
6	31	17	5.5	2	5.8	2	36	0.52	1	0.29
7	17	42	5.8	2	5.9	1.8	2.9	0.46	1	0.25
8	17	39	5.8	2	5.7	2.4	57	0.8	1	0.44
9	1	9	3.2	2.4	6	1.1	36	6.1	1	3.3
10	9	15	6	1.1	6.4	1.7	98	1.5	1	0.83
11	15	33	6.4	1.7	6.7	1.9	0.48	0.54	1	0.3
12	33	2	6.7	1.9	7.1	1.8	52	0.83	1	0.45
13	2	8	7.1	1.8	7.4	1.8	18	0.61	1	0.33
14	8	4	7.4	1.8	7.2	1.5	27	0.66	1	0.36
15	4	6	7.2	1.5	7.6	1.4	7.8	0.74	1	0.41
16	4	12	7.2	1.5	7.1	1.1	18	0.8	1	0.44
17	1	10	3.2	2.4	5.9	0.66	57	6.3	1	3.5
18	10	3	5.9	0.66	6.2	0.78	11	0.68	1	0.38
19	3	29	6.2	0.78	7	2.7	36	4.1	1	2.3
20	29	25	7	2.7	7	2.8	24	0.31	1	0.17
21	29	43	7	2.7	7.8	2.4	36	1.7	1	0.95
22	1	36	3.2	2.4	6	3.3	17	6.6	1	3.7
23	1	11	3.2	2.4	5.9	3.6	24	7.2	1	4
24	11	14	5.9	3.6	6.3	4.1	32	1.3	1	0.69
25	14	41	6.3	4.1	6.1	4.3	63	0.64	1	0.35
26	41	16	6.1	4.3	6.1	4.7	2.3	0.88	1	0.48
27	14	44	6.3	4.1	7	3.9	300	1.3	3	1.1
28	44	45	7	3.9	7.5	3.6	1.7	1.1	1	0.62
29	44	5	7	3.9	7	4.6	28	1.4	1	0.78
30	5	13	7	4.6	7.3	5.8	26	2.4	1	1.3
31	13	21	7.3	5.8	7.1	6.2	36	0.96	1	0.53
32	13	32	7.3	5.8	7.8	6.2	23	1.3	1	0.74
33	32	35	7.8	6.2	8.1	5.5	15	1.5	1	0.85
34	21	20	7.1	6.2	6.4	6.1	28	1.5	1	0.85
35	1	40	3.2	2.4	9.3	5.9	71	15	1	8.5
36	40	30	9.3	5.9	9.3	7.5	42	3.4	1	1.9
37	30	34	9.3	7.5	9.6	8.4	44	1.7	1	0.96
38	34	37	9.6	8.4	9.3	8.6	73	0.69	1	0.38
39	30	38	9.3	7.5	8.4	7.9	56	2.1	1	1.1
40	1	26	3.2	2.4	11	5	59	20	1	11
41	1	28	3.2	2.4	9.1	3.9	111	13	1	6.9
42	28	27	9.1	3.9	10	4.1	133	2.2	1	1.2
43	28	7	9.1	3.9	12	4.8	70	15	1	8.3
44	7	18	12	4.8	14	6.1	55	3.8	1	2.1
45	24	20	4.1	0.87	6.4	6.1	111	4.4	4	17
46	39	26	5.7	2.4	11	5	288	5.6	4	27
47	36	18	6	3.3	14	6.1	411	6	4	31
48	43	38	7.8	2.4	8.4	7.9	200	7.8	4	53
49	12	24	7.1	1.1	4.1	0.87	100	24	4	500

Table A.13: Case 4 : Christmas - 3 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	22	3.2	2.4	4.1	1.5	111	2.4	1	1.3
2	22	24	4.1	1.5	4.1	0.87	76	1.3	1	0.7
3	1	23	3.2	2.4	3.5	0.57	58	3.6	1	2
4	23	19	3.5	0.57	5.3	1.6	80	4.1	1	2.3
5	19	31	5.3	1.6	5.5	2	49	0.91	1	0.5
6	1	39	3.2	2.4	5.7	2.4	222	4.9	1	2.7
7	39	17	5.7	2.4	5.8	2	133	0.8	1	0.44
8	17	42	5.8	2	5.9	1.8	9.9	0.46	1	0.25
9	1	9	3.2	2.4	6	1.1	59	6.1	1	3.3
10	9	15	6	1.1	6.4	1.7	100	1.5	1	0.83
11	15	33	6.4	1.7	6.7	1.9	1.4	0.54	1	0.3
12	33	2	6.7	1.9	7.1	1.8	46	0.83	1	0.45
13	2	8	7.1	1.8	7.4	1.8	19	0.61	1	0.33
14	8	4	7.4	1.8	7.2	1.5	38	0.66	1	0.36
15	4	6	7.2	1.5	7.6	1.4	5.2	0.74	1	0.41
16	4	12	7.2	1.5	7.1	1.1	22	0.8	1	0.44
17	1	10	3.2	2.4	5.9	0.66	44	6.3	1	3.5
18	10	3	5.9	0.66	6.2	0.78	9.2	0.68	1	0.38
19	3	29	6.2	0.78	7	2.7	93	4.1	1	2.3
20	29	25	7	2.7	7	2.8	79	0.31	1	0.17
21	1	36	3.2	2.4	6	3.3	44	6.6	1	3.7
22	36	11	6	3.3	5.9	3.6	82	0.57	1	0.31
23	36	43	6	3.3	7.8	2.4	177	4	1	2.2
24	1	14	3.2	2.4	6.3	4.1	65	8.2	1	4.5
25	14	41	6.3	4.1	6.1	4.3	166	0.64	1	0.35
26	41	16	6.1	4.3	6.1	4.7	13	0.88	1	0.48
27	14	44	6.3	4.1	7	3.9	211	1.3	1	0.74
28	44	45	7	3.9	7.5	3.6	2.9	1.1	1	0.62
29	44	5	7	3.9	7	4.6	80	1.4	1	0.78
30	5	13	7	4.6	7.3	5.8	133	2.4	1	1.3
31	13	21	7.3	5.8	7.1	6.2	96	0.96	1	0.53
32	13	32	7.3	5.8	7.8	6.2	100	1.3	1	0.74
33	32	35	7.8	6.2	8.1	5.5	67	1.5	1	0.85
34	21	20	7.1	6.2	6.4	6.1	85	1.5	1	0.85
35	35	40	8.1	5.5	9.3	5.9	300	2.7	3	2.2
36	45	28	7.5	3.6	9.1	3.9	211	3.4	1	1.9
37	28	27	9.1	3.9	10	4.1	177	2.2	1	1.2
38	1	30	3.2	2.4	9.3	7.5	133	17	1	9.5
39	30	34	9.3	7.5	9.6	8.4	177	1.7	1	0.96
40	34	37	9.6	8.4	9.3	8.6	411	0.69	4	0.61
41	30	38	9.3	7.5	8.4	7.9	188	2.1	1	1.1
42	34	26	9.6	8.4	11	5	188	7.4	1	4.1
43	26	7	11	5	12	4.8	244	2.7	2	1.9
44	7	18	12	4.8	14	6.1	244	3.8	2	2.7
45	24	25	4.1	0.87	7	2.8	84	1.7	4	2.6
46	31	43	5.5	2	7.8	2.4	277	2.8	4	7.1
47	42	27	5.9	1.8	10	4.1	311	3.6	4	11
48	12	18	7.1	1.1	14	6.1	411	20	4	344

Table A.14: Case 4 : Summer - 3 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	22	3.2	2.4	4.1	1.5	111	2.4	1	1.3
2	22	24	4.1	1.5	4.1	0.87	89	1.3	1	0.7
3	24	23	4.1	0.87	3.5	0.57	93	1.4	1	0.76
4	1	19	3.2	2.4	5.3	1.6	133	4.4	1	2.4
5	19	31	5.3	1.6	5.5	2	27	0.91	1	0.5
6	1	39	3.2	2.4	5.7	2.4	70	4.9	1	2.7
7	39	17	5.7	2.4	5.8	2	45	0.8	1	0.44
8	17	42	5.8	2	5.9	1.8	4.4	0.46	1	0.25
9	42	15	5.9	1.8	6.4	1.7	122	1.1	1	0.61
10	15	33	6.4	1.7	6.7	1.9	0.6	0.54	1	0.3
11	33	2	6.7	1.9	7.1	1.8	61	0.83	1	0.45
12	2	8	7.1	1.8	7.4	1.8	22	0.61	1	0.33
13	8	4	7.4	1.8	7.2	1.5	33	0.66	1	0.36
14	4	6	7.2	1.5	7.6	1.4	9.4	0.74	1	0.41
15	1	9	3.2	2.4	6	1.1	45	6.1	1	3.3
16	9	3	6	1.1	6.2	0.78	13	0.81	1	0.45
17	3	10	6.2	0.78	5.9	0.66	72	0.68	1	0.38
18	3	12	6.2	0.78	7.1	1.1	21	1.9	1	1.1
19	1	29	3.2	2.4	7	2.7	46	7.5	1	4.2
20	1	36	3.2	2.4	6	3.3	26	6.6	1	3.7
21	36	11	6	3.3	5.9	3.6	28	0.57	1	0.31
22	11	14	5.9	3.6	6.3	4.1	36	1.3	1	0.69
23	14	41	6.3	4.1	6.1	4.3	79	0.64	1	0.35
24	41	16	6.1	4.3	6.1	4.7	2.6	0.88	1	0.48
25	14	44	6.3	4.1	7	3.9	377	1.3	3	1.1
26	44	45	7	3.9	7.5	3.6	2.8	1.1	1	0.62
27	44	5	7	3.9	7	4.6	34	1.4	1	0.78
28	36	25	6	3.3	7	2.8	38	2.3	1	1.3
29	25	43	7	2.8	7.8	2.4	45	1.7	1	0.95
30	5	13	7	4.6	7.3	5.8	30	2.4	1	1.3
31	13	21	7.3	5.8	7.1	6.2	45	0.96	1	0.53
32	1	20	3.2	2.4	6.4	6.1	47	12	1	6.7
33	20	32	6.4	6.1	7.8	6.2	29	2.9	1	1.6
34	32	35	7.8	6.2	8.1	5.5	19	1.5	1	0.85
35	35	40	8.1	5.5	9.3	5.9	78	2.7	1	1.5
36	40	30	9.3	5.9	9.3	7.5	52	3.4	1	1.9
37	30	34	9.3	7.5	9.6	8.4	63	1.7	1	0.96
38	34	37	9.6	8.4	9.3	8.6	400	0.69	4	0.61
39	1	38	3.2	2.4	8.4	7.9	87	24	1	13
40	1	28	3.2	2.4	9.1	3.9	111	13	1	6.9
41	28	27	9.1	3.9	10	4.1	133	2.2	1	1.2
42	28	26	9.1	3.9	11	5	81	14	1	7.5
43	26	7	11	5	12	4.8	111	2.7	1	1.5
44	7	18	12	4.8	14	6.1	70	3.8	1	2.1
45	23	21	3.5	0.57	7.1	6.2	144	1.2	4	1.3
46	31	38	5.5	2	8.4	7.9	200	2.2	4	4.4
47	29	37	7	2.7	9.3	8.6	477	4.9	4	22
48	6	18	7.6	1.4	14	6.1	411	7.2	4	45
49	12	23	7.1	1.1	3.5	0.57	111	25	4	577

Table A.15: Case 4 : New year's eve - 3 Edges

Links	Sub1	Sub2	X1	Y1	X2	Y2	Peaks (kW)	Distance (km)	Cables	Cost (M Dkk)
1	1	22	3.2	2.4	4.1	1.5	177	2.4	1	1.3
2	22	24	4.1	1.5	4.1	0.87	133	1.3	1	0.7
3	1	23	3.2	2.4	3.5	0.57	100	3.6	1	2
4	23	19	3.5	0.57	5.3	1.6	144	4.1	1	2.3
5	19	31	5.3	1.6	5.5	2	67	0.91	1	0.5
6	31	17	5.5	2	5.8	2	199	0.52	1	0.29
7	1	39	3.2	2.4	5.7	2.4	366	4.9	3	4.1
8	39	42	5.7	2.4	5.9	1.8	7.2	1.2	1	0.68
9	42	15	5.9	1.8	6.4	1.7	177	1.1	1	0.61
10	1	9	3.2	2.4	6	1.1	111	6.1	1	3.3
11	9	33	6	1.1	6.7	1.9	2.6	2	1	1.1
12	33	2	6.7	1.9	7.1	1.8	82	0.83	1	0.45
13	2	8	7.1	1.8	7.4	1.8	31	0.61	1	0.33
14	8	4	7.4	1.8	7.2	1.5	52	0.66	1	0.36
15	4	6	7.2	1.5	7.6	1.4	17	0.74	1	0.41
16	4	12	7.2	1.5	7.1	1.1	40	0.8	1	0.44
17	1	3	3.2	2.4	6.2	0.78	17	6.8	1	3.7
18	1	10	3.2	2.4	5.9	0.66	75	6.3	1	3.5
19	10	29	5.9	0.66	7	2.7	133	4.6	1	2.5
20	29	25	7	2.7	7	2.8	86	0.31	1	0.17
21	29	43	7	2.7	7.8	2.4	233	1.7	1	0.95
22	25	36	7	2.8	6	3.3	62	2.3	1	1.3
23	36	11	6	3.3	5.9	3.6	100	0.57	1	0.31
24	11	14	5.9	3.6	6.3	4.1	133	1.3	1	0.69
25	1	41	3.2	2.4	6.1	4.3	200	8.5	1	4.7
26	41	16	6.1	4.3	6.1	4.7	13	0.88	1	0.48
27	16	5	6.1	4.7	7	4.6	166	1.9	1	1
28	5	44	7	4.6	7	3.9	377	1.4	3	1.2
29	44	45	7	3.9	7.5	3.6	5.4	1.1	1	0.62
30	5	13	7	4.6	7.3	5.8	200	2.4	1	1.3
31	13	21	7.3	5.8	7.1	6.2	144	0.96	1	0.53
32	13	32	7.3	5.8	7.8	6.2	166	1.3	1	0.74
33	32	35	7.8	6.2	8.1	5.5	97	1.5	1	0.85
34	21	20	7.1	6.2	6.4	6.1	122	1.5	1	0.85
35	35	40	8.1	5.5	9.3	5.9	377	2.7	3	2.2
36	45	28	7.5	3.6	9.1	3.9	344	3.4	3	2.8
37	28	27	9.1	3.9	10	4.1	344	2.2	3	1.8
38	1	30	3.2	2.4	9.3	7.5	188	17	1	9.5
39	30	34	9.3	7.5	9.6	8.4	200	1.7	1	0.96
40	34	37	9.6	8.4	9.3	8.6	500	0.69	4	0.61
41	30	38	9.3	7.5	8.4	7.9	222	2.1	1	1.1
42	34	26	9.6	8.4	11	5	288	7.4	2	5.3
43	26	7	11	5	12	4.8	188	2.7	1	1.5
44	7	18	12	4.8	14	6.1	377	3.8	3	3.1
45	24	3	4.1	0.87	6.2	0.78	18	1.8	4	3
46	17	14	5.8	2	6.3	4.1	133	4	4	14
47	15	27	6.4	1.7	10	4.1	311	6.6	4	38
48	12	18	7.1	1.1	14	6.1	411	20	4	344

Table A.16: Case 4 : January - 3 Edges

Appendix B

Plots

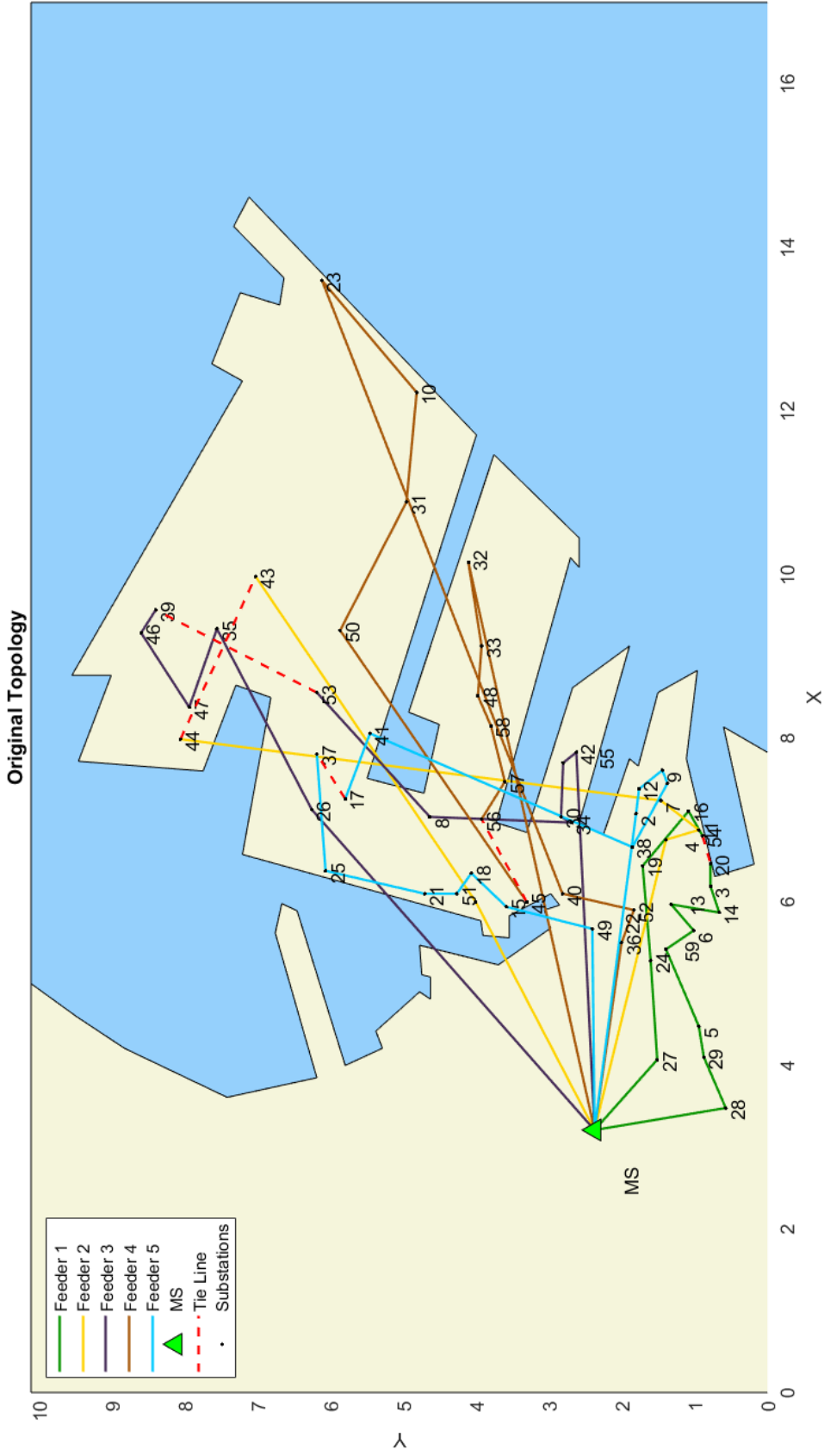


Figure B.1



Figure B.2

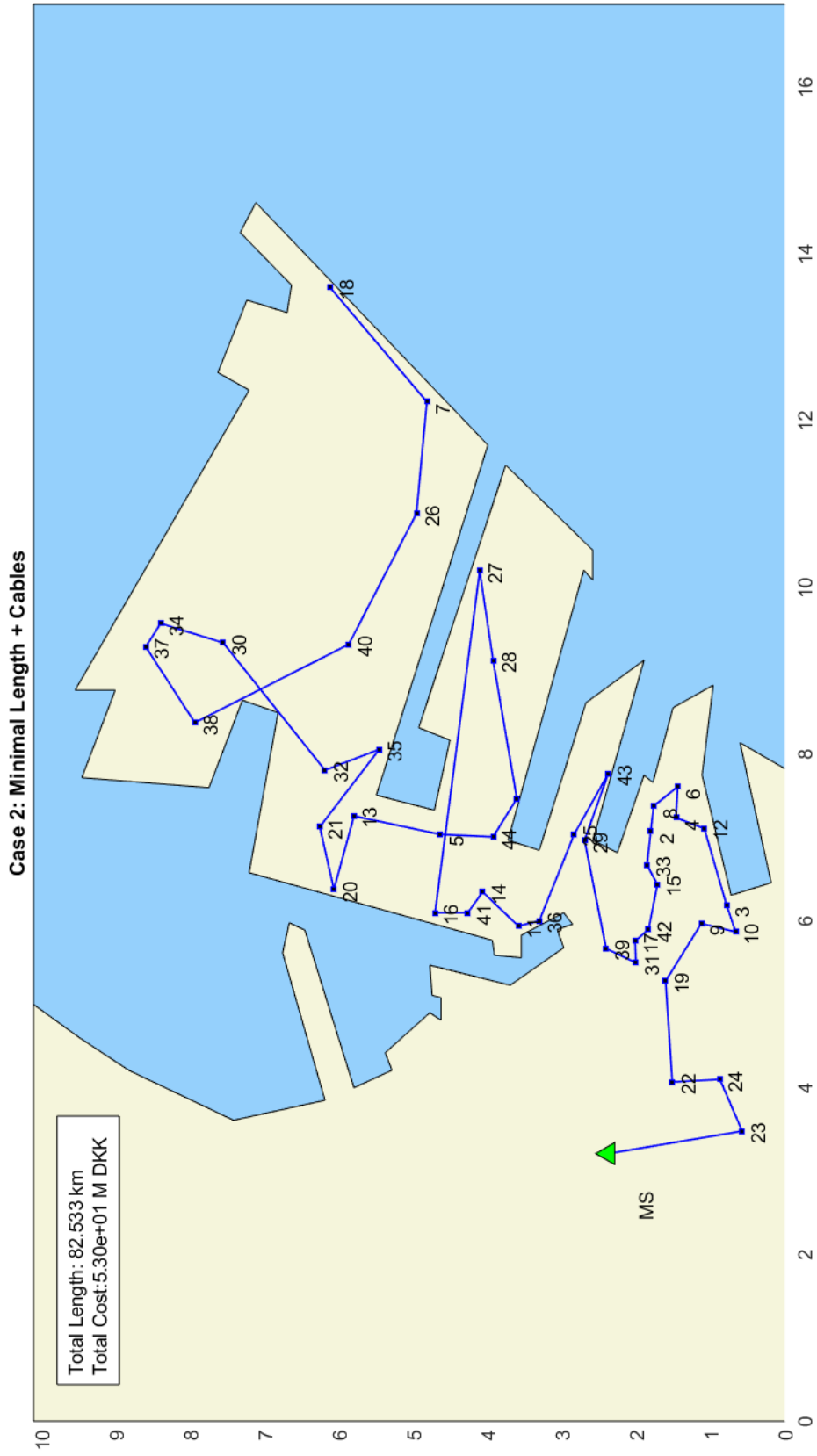


Figure B.3

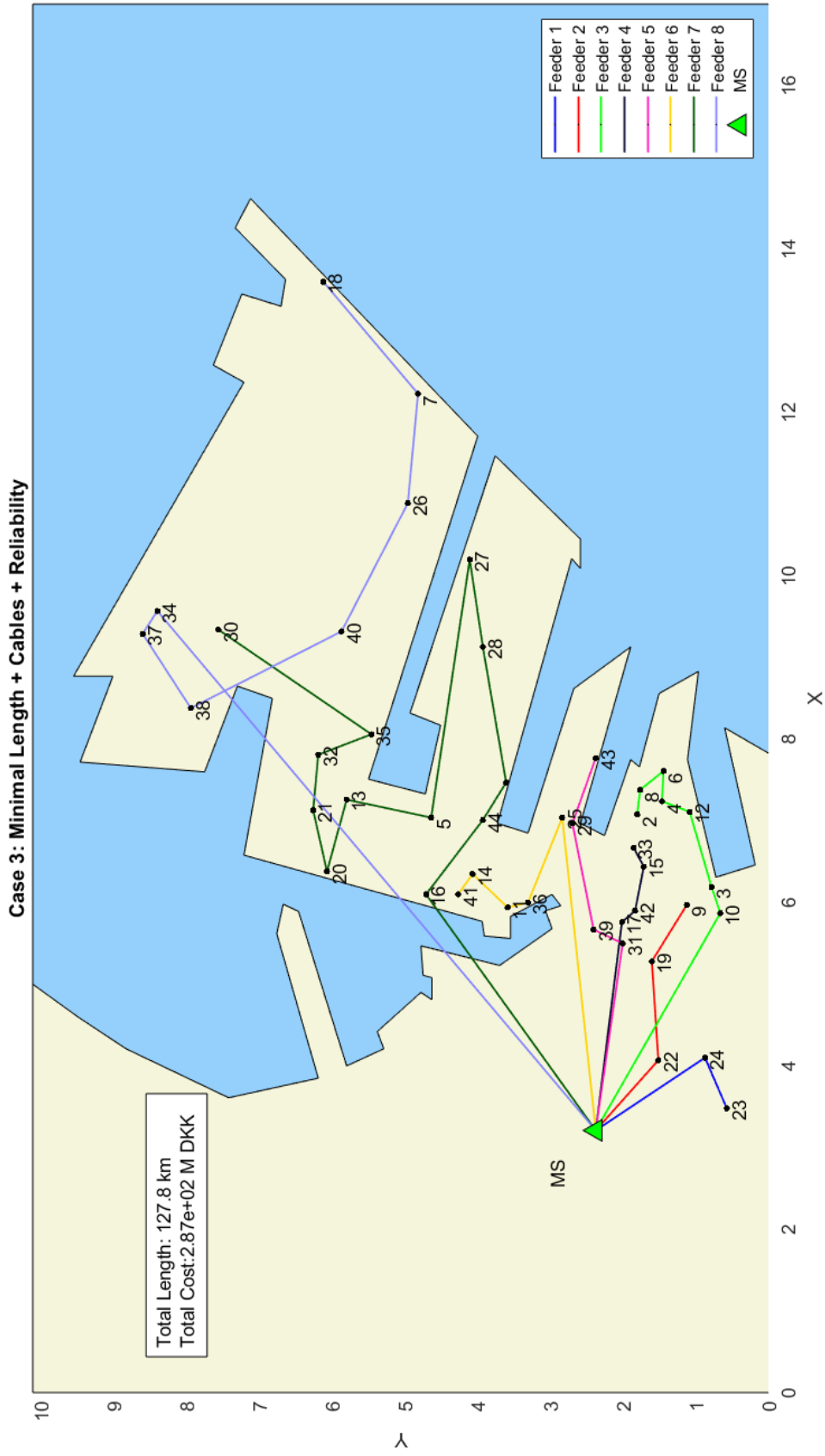


Figure B.4

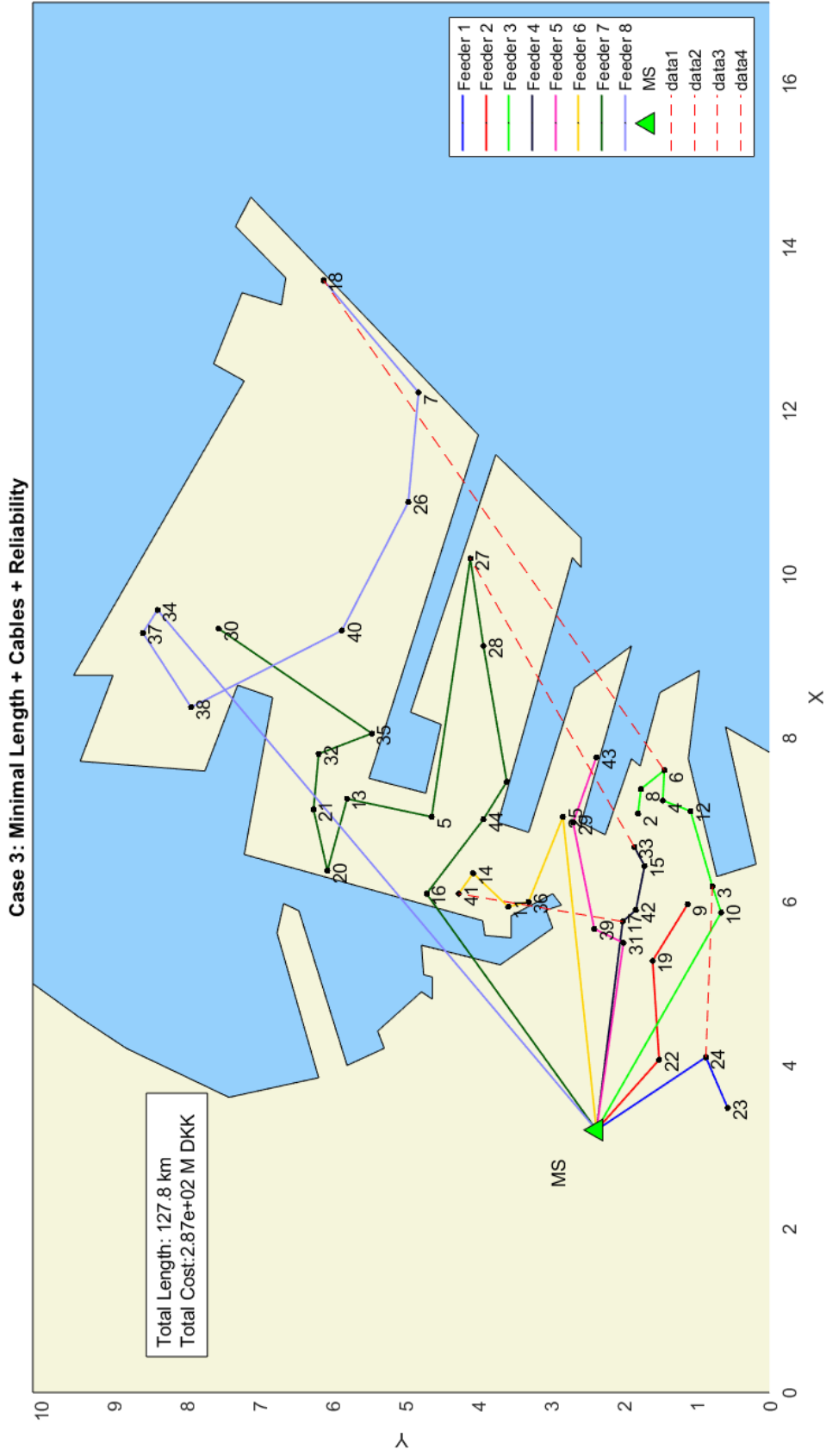


Figure B.5

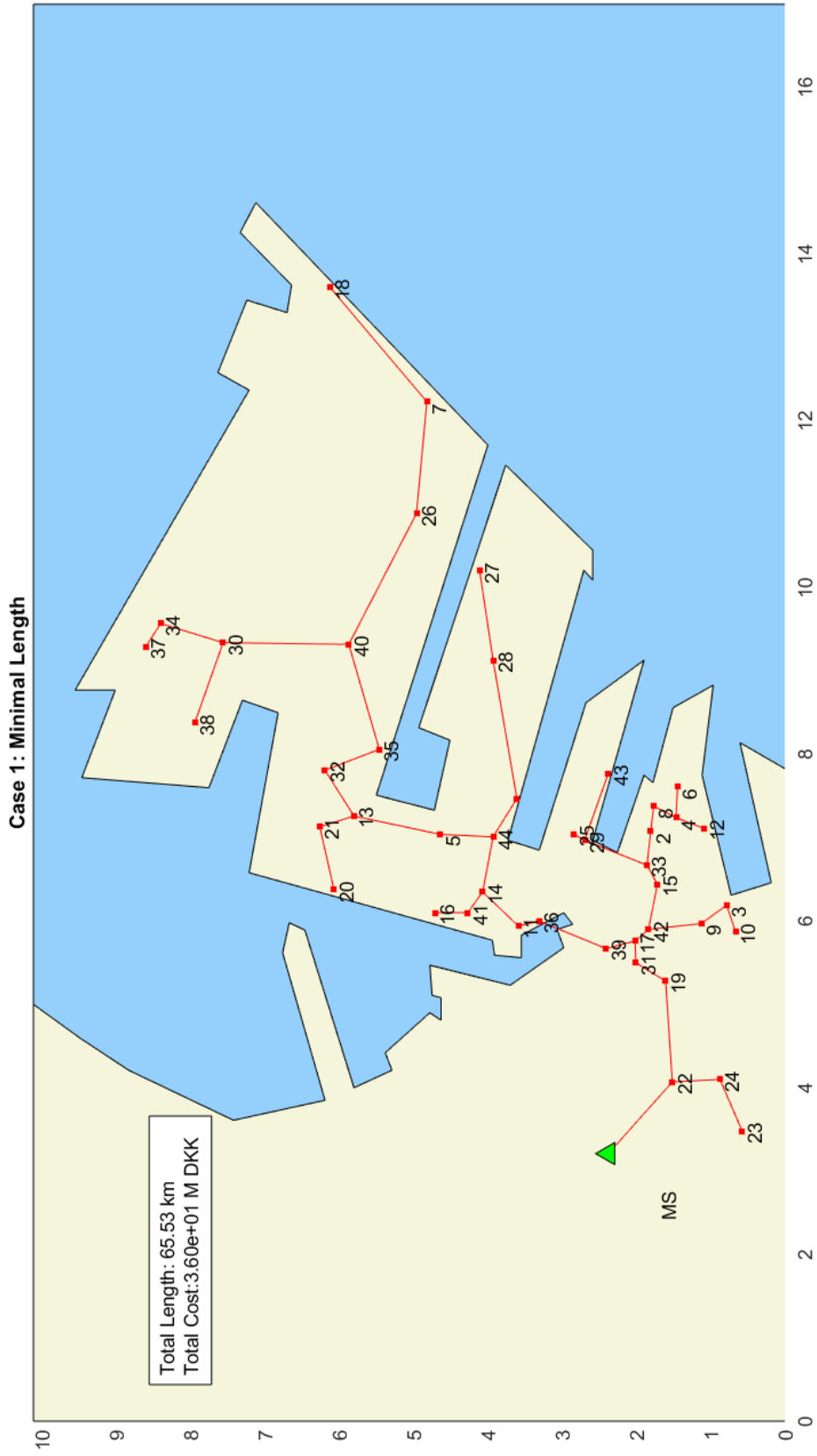


Figure B.6

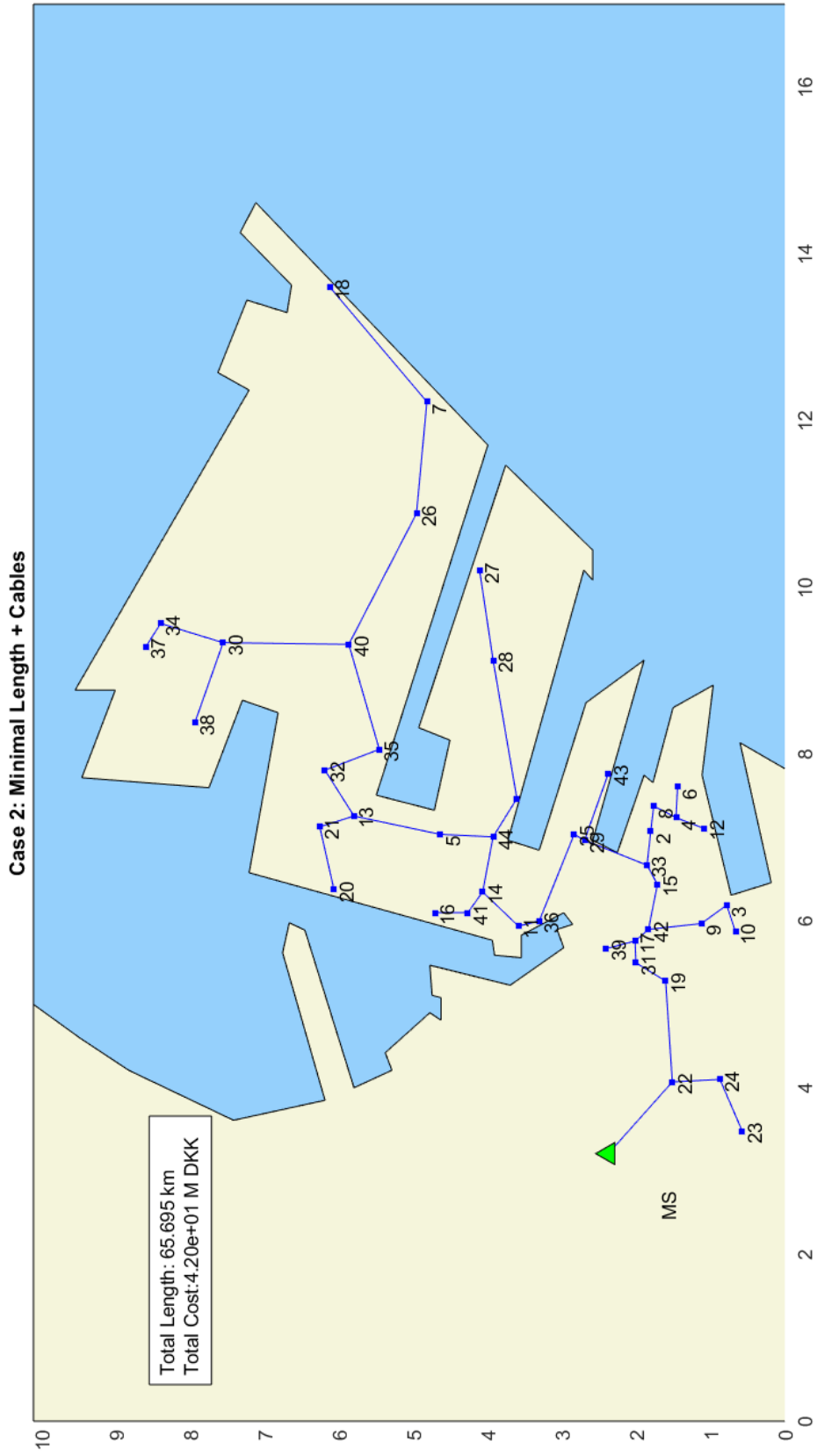


Figure B.7

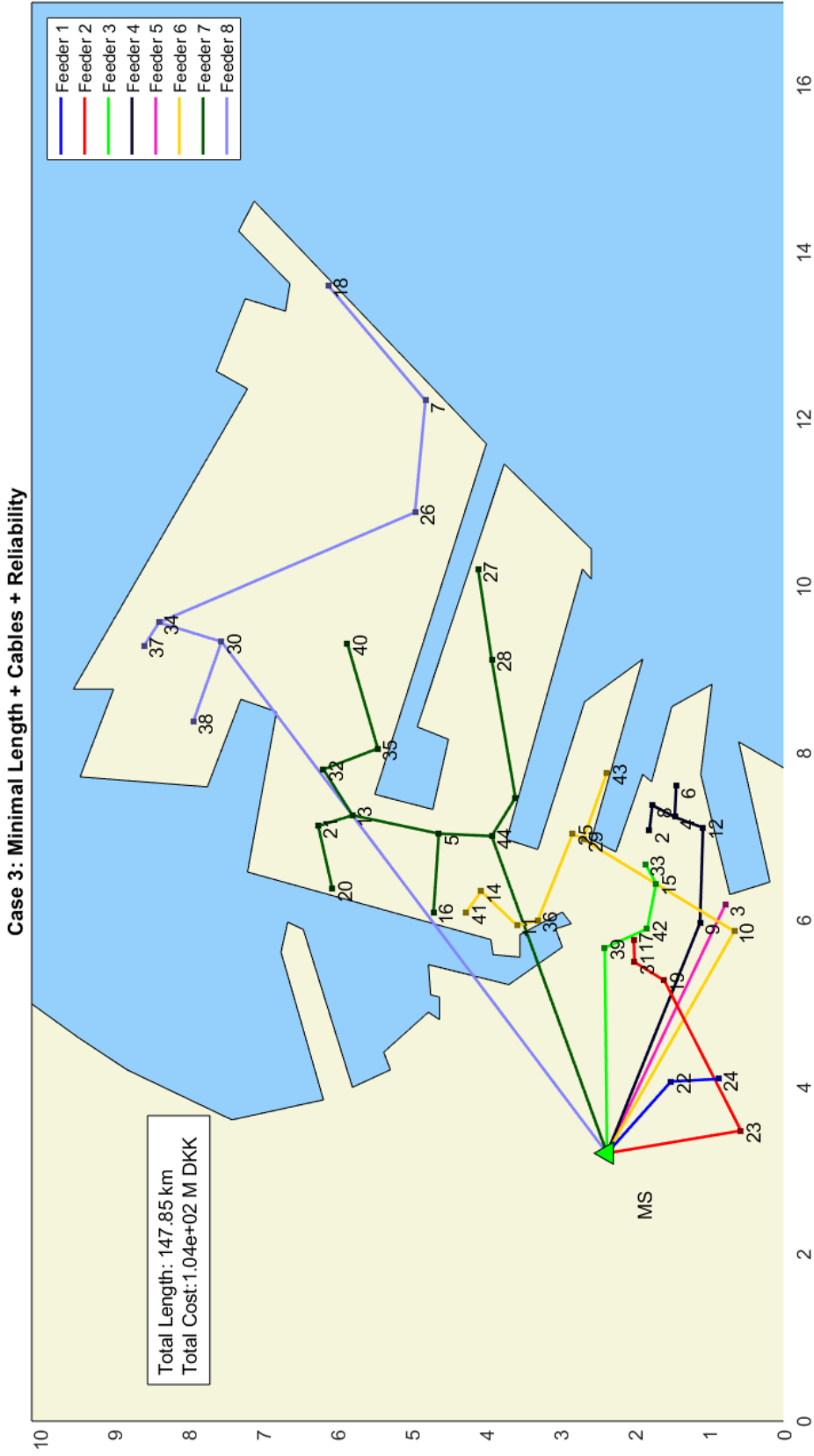


Figure B.8

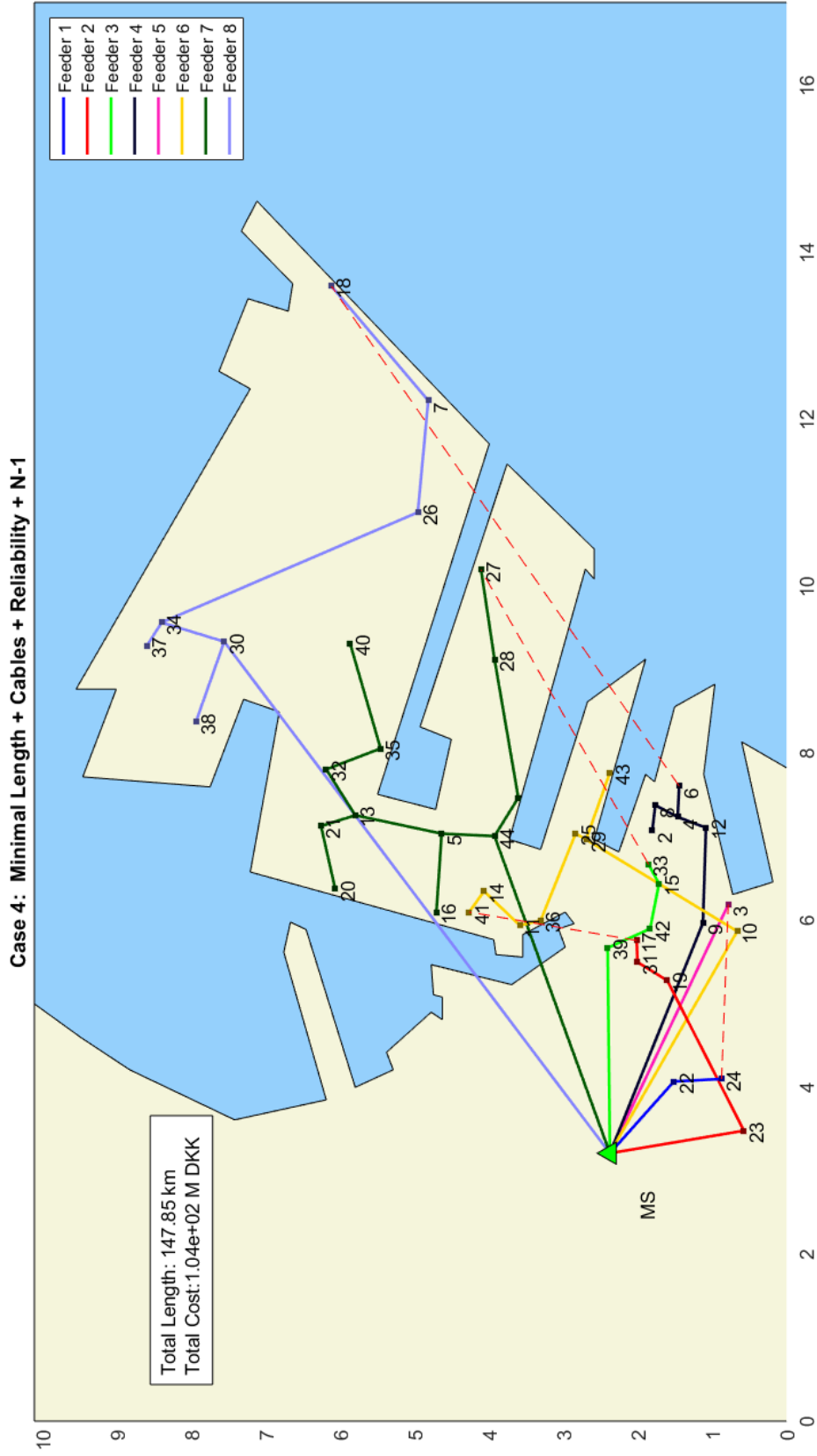


Figure B.9

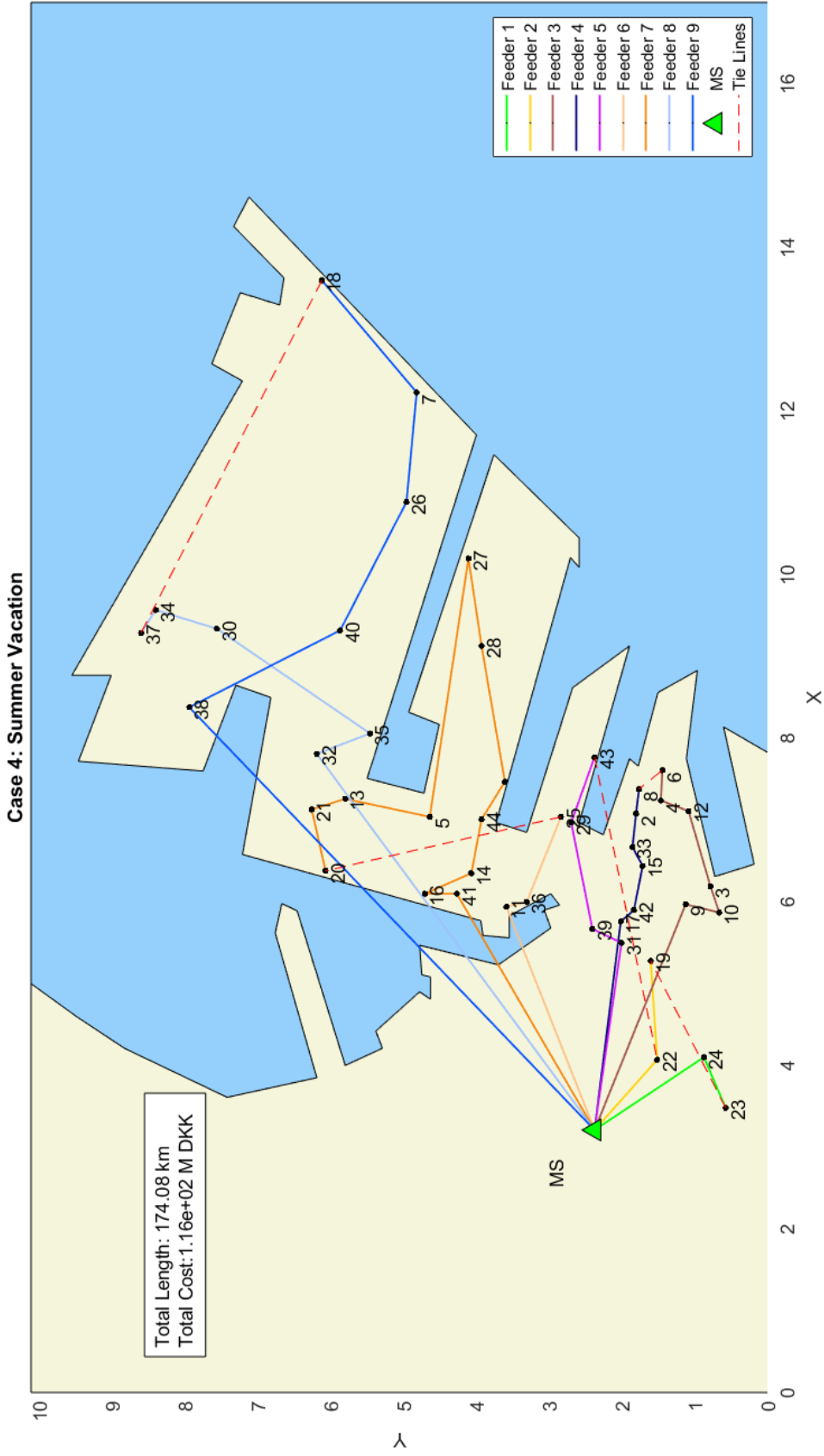


Figure B.10

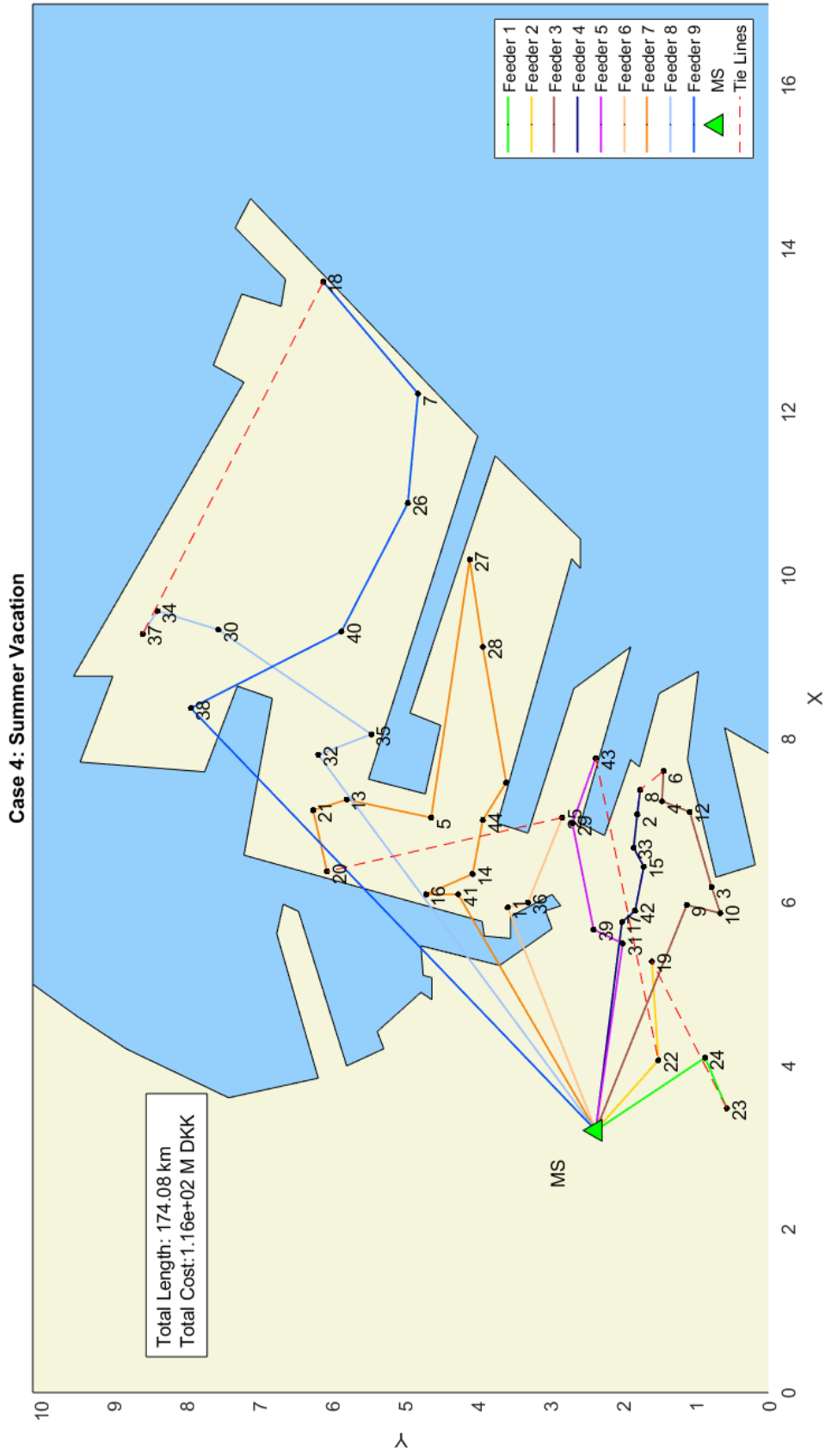


Figure B.11

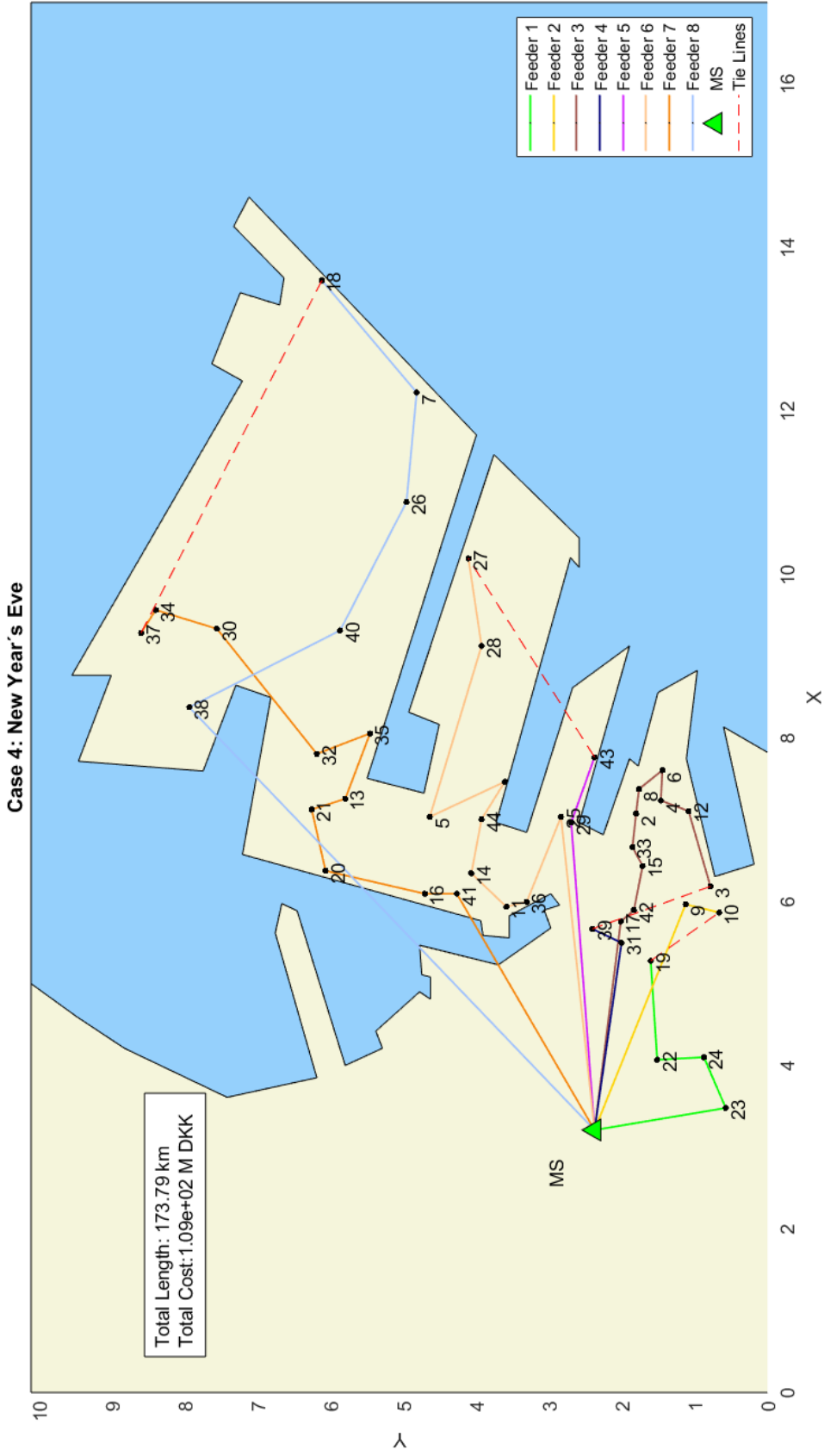


Figure B.12

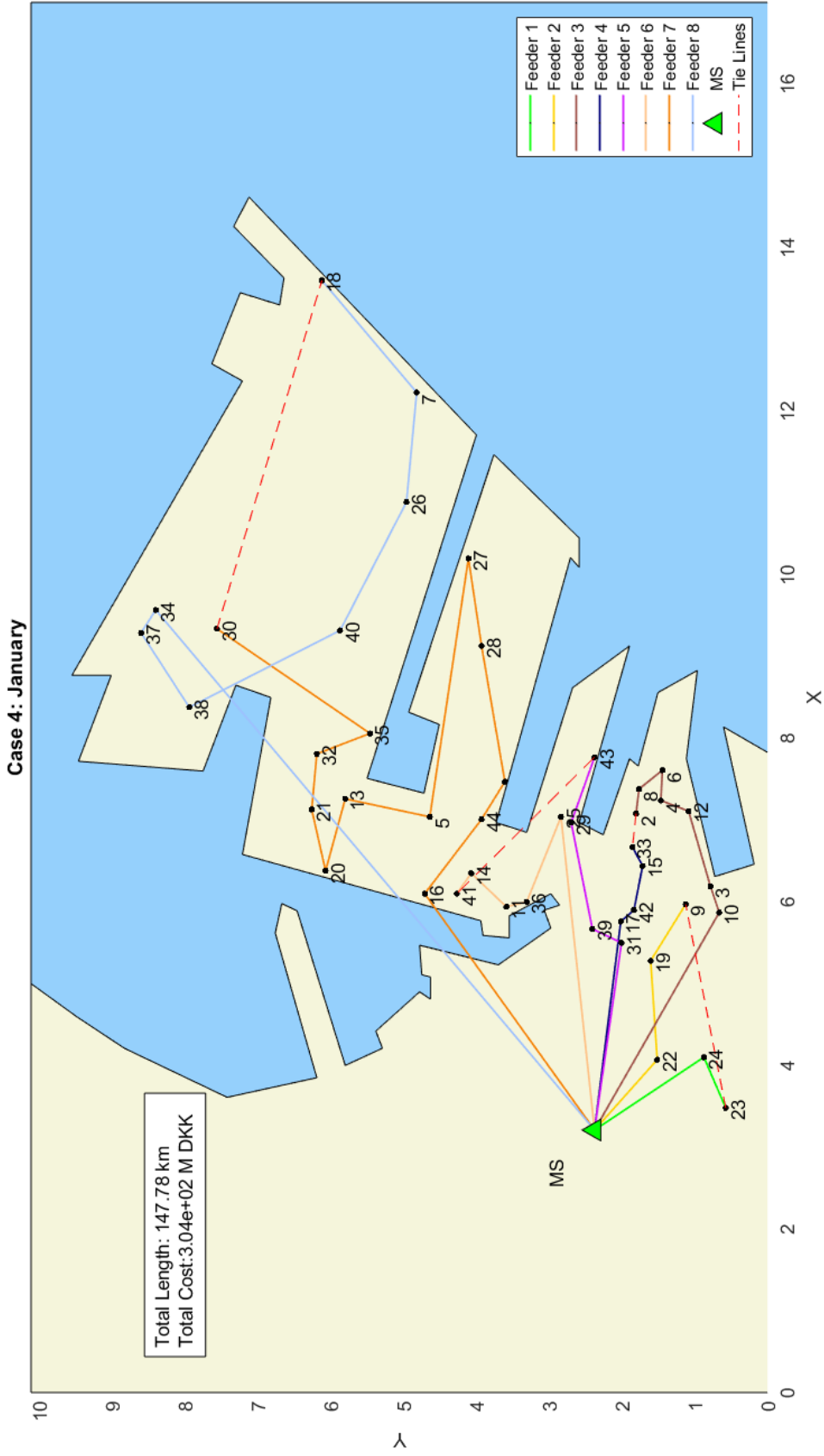


Figure B.13